TELEMATIC SCHOOLS PROJECT

2025 SUBJECT WORKBOOK
Grade 12

$f(x)=x^2$ MATHEMATICS

A joint initiative between the Western Cape Education Department and Stellenbosch University.

Western Cape Government Education FOR YOU

Stellenbosch UNIVERSITY IYUNIVESITHI UNIVERSITEIT

INTRODUCTION

Dear Grade 12 Learner

In 2025 there will be 10 live Mathematics Telematics sessions, 5 of these sessions are in English and 5 in Afrikaans. Some of these sessions are on grade 12 content and others on grade 11. All grade 11 content will be assessed in the grade 12 examination.

This workbook contain material for all the sessions Grade 12 learners can benefit from.

Please email, school@sun.ac.za with your name, surname, email and school name to register to obtain a username and password. You will then be able to access all previously recorded Telematics sessions and materials on other Topics from the Telematics Schools Project website, https://schools.sun.ac.za/ . Please note that accessing these recordings from this website is zero-rated.

In the Telematic lesson, the presenters will provide you with a summary of the important concepts and together with you they will work though some of the exercises. You are encouraged to come prepared, have a pen and enough paper (ideally a hard cover exercise book) and a scientific calculator.

You are also encouraged to participate fully in the lesson by asking questions and working out the exercises, and where you are asked to do so, sms or e-mail your answers to the studio.

Remember:" Success is not an event, it is the result of regular and consistent hard work".

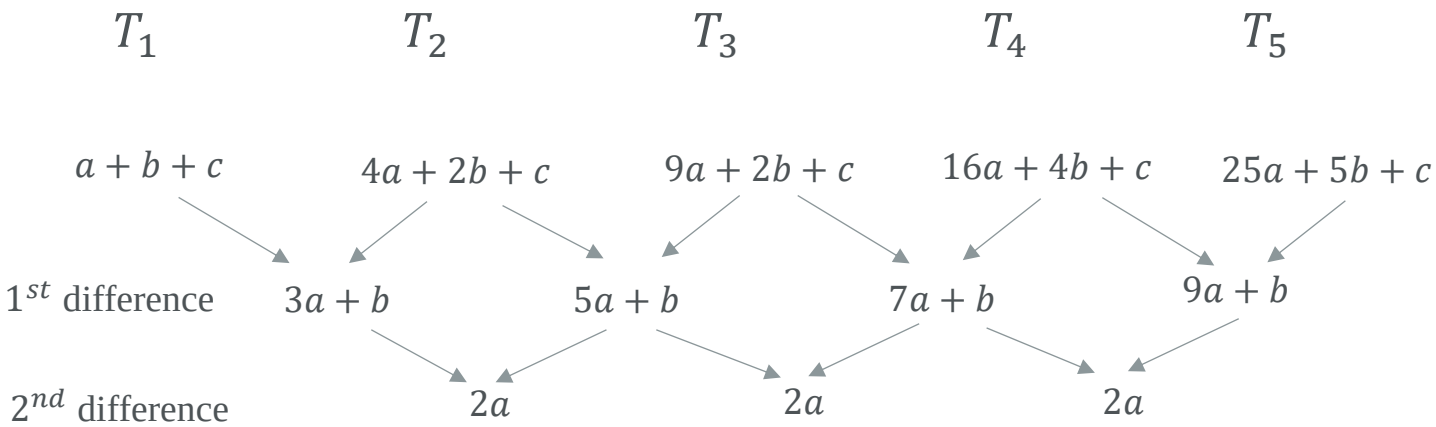
GOODLUCK, Wishing you all the success you deserve!

Session/Sessie	Date/Datum	Grade	Time/Tyd	Topic/Onderwerp
1.	19/02/2025	Gr 12	15h00-16h00	Sequences & Series/functions
2.	20/02/2025	Gr 12	15h00-16h00	Rye en Reekse
3.	20/05/2025	Gr 11 & Gr 12	15h00-16h00	Analytical Geometry
4.	21/05/2025	Gr 11 & Gr 12	15h00-16h00	Analitiese Meetkunde
5.	02/06/2025	Gr 12	15h00-16h00	Cubic Graphs
6.	03/06/2025	Gr 12	15h00-16h00	Kubieka Grafiek
7.	25/08/2025	Gr 11 & Gr 12	16h00-17h00	Statistics
8.	26/08/2025	Gr 11 & Gr 12	16h00-17h00	Statistiek
9.	22/10/2025	Gr 11 & Gr 12	15h00-16h00	Paper 1: Functions
10.	23/10/2025	Gr 11 & Gr 12	15h00-16h00	Vraestel 1: Funksies

SESSION 1: Number Patterns, Sequences and Series

Quadratic Sequence/Series

Consider a quadratic sequence with terms $T_1; T_2; T_3; T_4; T_5; \ldots\ldots$



Arithmetic Sequence/Series	**Geometric Sequence/Series**	**Quadratic Sequence/Series**
$d = T_2 - T_1$ $d = T_n - T_{n-1}$	$r = \frac{T_2}{T_1}$ $r = \frac{T_n}{T_{n-1}}$	1^{st} difference $3a+b;\ 5a+b;\ 7a+b$ 2^{nd} difference$= 2a$
$T_n = a + (n-1)d$	$T_n = ar^{n-1}$	$T_n = an^2 + bn + c$ $T_1 = a + b + c$
$S_n = \frac{n}{2}[2a + (n-1)d]$	$S_n = \frac{a(r^n - 1)}{r - 1}$	
	Infinite Sequence/Series	
	Convergent if: $-1 < r < 1$	
	$S_\infty = \frac{a}{1-r}$	

SESSION 1 continued

Example 1:

Given the arithmetic series: $2 + 9 + 16 + \ldots\ldots$ (to 251 terms)

1.1 Write down the fourth term of the series. (1)

1.2 Calculate the 251^{st} term of the series (3)

1.3 Express the series is sigma notation. (2)

1.4 Calculate the sum of the series. (2)

Example 2:

The first four terms of a quadratic number pattern are: $-1\ ;\ 2\ ;\ 9\ ;\ 20$.

2.1 Determine the general term of the quadratic number pattern. (4)

2.2 Calculate the value of the 48^{th} term of the quadratic number pattern (2)

2.3 Show that the sum of the first differences of this quadratic number pattern can be given by $S_n = 2n^2 + n$ (3)

Example 3: Given $\sum_{p=4}^{21}(-3)^p$

3.1 Write down the values of the first three terms of the series. (2)

3.2 Write down the value of the constant ratio. (1)

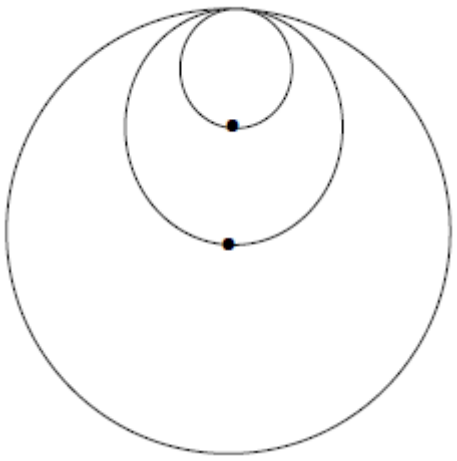
3.3 Will the given series converge? Explain you answer. (2)

3.4 How can you change the question for it to be a converging series? (1)

Example 4:

A circle with radius 6cm is drawn. A second, smaller circle is drawn through the centre of the first circle and also touches the first circle internally, as shown in the diagram.

A third , smaller circle is drawn through the centre of the second circle and touches the second circle internally. The process of drawing circles continues and forms a geometric pattern.



4.1 Write down the radius of the 3^{rd} circle. (2)

4.2 Calculate the sum of the areas of the first 10 circles. (4)

4.3 Which circle has a diameter of $\frac{3}{128}$ cm ? (4)

SESSION 1: EXERCISE

1. Given the following quadratic sequence: -2 ; 0 ; 3 ; 7 ; ….

1.1 Write down the value of the next term of this sequence. (1)

1.2 Determine the expression for the n^{th} term of this sequence. (5)

1.3 Which term of the sequence will be equal to 322? (4)

2. The first three terms of an arithmetic sequence are: 4 ; 13 and 22

2.1 Write down the fourth term of this sequence. (1)

2.2 Determine the general term of the sequence. (2)

2.3 Consider the terms of the series which are even. Calculate the sum of the first 25 terms which are even. (4)

2.4 The original sequence (4 ; 13 and 22) forms the first difference of a new sequence with a first term equal to -6. Determine a formula for the n^{th} term of this new sequence. (4)

3. Consider the infinite geometric series: 45 + 40,5 + 36,45 + ….

3.1 Calculate the value of the TWELFTH term of the series (correct to TWO decimal places). (3)

3.2 Explain why this series converges. (1)

3.3 Calculate the sum to infinity of the series. (2)

4. The sequence of the first difference of a quadratic pattern is: 1; 3; 5; ……

4.1 If T_{99} of this quadratic pattern is 9 632 , calculate the value of T_{98}. (3)

4.2 If it is further given that the third term of the quadratic pattern is 32 , determine the general term T_n , of the quadratic pattern. (5)

5. Calculate the value of: $\sum_{k=1}^{50}(100 - 3k)$ (4)

SESSION 1: EXERCISE continued

6. **Consider the series: $S_n = -3 + 5 + 13 + 21 + \ldots\ldots$ to n terms .**

Write S_n in sigma notation (4)

7. Given the arithmetic series: $a + 13 + b + 27 + \ldots$

7.1 Show that $a = 6$ and $b = 20$ (2)

7.2 Calculate the sum of the first 20 terms of the series. (3)

7.3 Write the series in the previous question in sigma notation. (2)

8. Given the geometric series: $(x - 2) + (x^2 - 4) + (x^3 + 2x^2 - 4x - 8) + \ldots$

8.1 Determine the values of x for which the series converges (4)

8.2 If $x = -\frac{3}{2}$, calculate the sum to infinity of the given series . (3)

9. The first term of an arithmetic series is 7. The common difference of this series is 5 and the series contains 20 terms.

9.1 Calculate the sum of the series. (2)

9.2 The original arithmetic series is extended to 75 terms. The sum of these 75 terms is 14400. Using sigma notation write down an equation for the sum of the terms added to the original series. (4)

10 Consider an arithmetic sequence which has the second term equal to 8 and the fifth term equal to 10 .

10.1 Determine the common difference of this sequence. (3)

10.2 Write down the sum of the first 50 terms of this sequence, using sigma notation. (2)

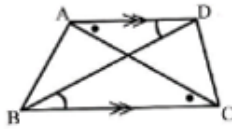
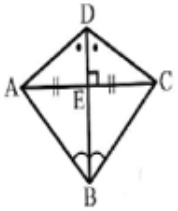
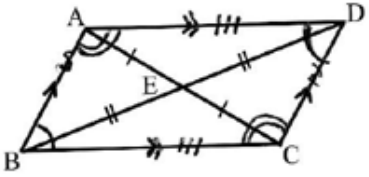
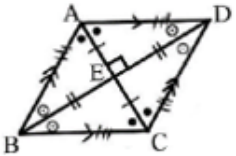
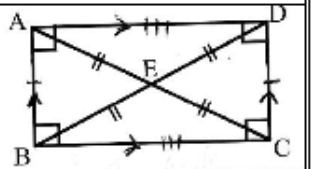
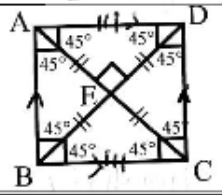
10.3 Determine the sum of the first 50 terms of the sequence. (3)

SESSION 3: ANALYTICAL GEOMETRY SUMMARY:

FORMULAE/CONCEPTS	Summary of procedure	EXAMPLE
Consider any two points $(x_1; y_1)$ and $(x_2; y_2)$ in a cartesian plane		Suppose $A(-3; -5)$ and $B\ (2;\ 3)$ are points in a cartesian plane.
Distance or Length of line Formula: $d = \sqrt{(x_2 - x_1)^2+(y_2 - y_1)^2}$	Label the points as point one and the other as point 2 and then substitute into the formula.	Determine the length AB: $d = \sqrt{(x_2 - x_1)^2+(y_2 - y_1)^2}$ $d_{AB} = \sqrt{((2) - (-3))^2+((3) - (-5))^2}$ $d_{AB} = \sqrt{(2 + 3))^2+(3 + 5)^2}$ $d_{AB} = \sqrt{(5))^2+(8)^2}$ $= \sqrt{89} = 9,43$ units
Midpoint of a line: $(\frac{x_2 + x_1}{2}; \frac{y_2 + y_1}{2})$	Label the points as point one and the other as point 2 and then substitute into the formula.	Determine the coordinates of E the midpoint of AB. $(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2})$ $E\left(\frac{-3+2}{2}; \frac{-5+3}{2}\right)$ $E\left(\frac{-1}{2}; \frac{-2}{2}\right) = E\left(\frac{-1}{2}; -1\right)$
Gradient: $m = \frac{y_2 - y_1}{x_2 - x_1}$	Label the points as point one and the other as point 2 and then substitute into the formula.	Determine the gradient of AB. $m = \frac{y_2-y_1}{x_2-x_1}$ $m_{AB} = \frac{(3)-(-5)}{(2)-(-3)} = \frac{3+5}{2+3} = \frac{8}{5}$
Parallel Lines: The gradients of the lines are equal i.e. $\boldsymbol{m_1 = m_2}$	Determine the gradient of one line then the parallel lines gradient is the same.	Determine the gradient of line CD if $CD \parallel AB$. $m_{AB} = \frac{8}{5}$ $\therefore m_{CD} = \frac{8}{5}$ because $CD \parallel AB$
Perpendicular Lines The product of the gradients is -1 i.e. $\boldsymbol{m_1 \times m_2 = -1}$	Take the reciprocal of the gradient of the given line and change the sign. Given $m_1 = +\frac{3}{}$ then $m_2 = -\frac{3}{}$	Determine the gradient of line CD if $CD \perp AB$. $m_{AB} = +\frac{8}{5}$ $\therefore m_{CD} = -\frac{5}{8}$ because $CD \perp AB$
Co-linear Points A, B and C $m_{AB} = m_{BC}$	Calculate the gradient of segments one point must be common. i.e. $m_{AB} = m_{BC}$	Determine wheth), $B\ (2;\ 3)$ and C $m_{BC} = \frac{y_3 - y_2}{x_3 - x_2}$ $m_{AB} = \frac{8}{5}$ $m_{BC} = \frac{(11-3)}{(7-2)} = \frac{8}{5}$ Hence A, B and C are collinear.

SESSION 3: ANALYTICAL GEOMETRY SUMMARY:

Gradient (inclination) of a line $\tan\theta = m_{AB}$ **θ is the angle of inclination** $m > 0 \rightarrow \quad \theta < 90°$ $\tan\theta = m_{AB}$ $m < 0 \rightarrow 90° < \theta < 180°$	Use calculator to determine $\tan^{-1}(m)$. Use the positive value of m. If m is negative, the angle you obtained must be subtracted from 180°.	(a) Determine the inclination of AB $m_{AB} = \frac{8}{5}$ **and** $\therefore \tan\theta = \frac{8}{5}$ **To get θ , arc(shift) tan** $\frac{8}{5}$, $\therefore \quad \theta = 57{,}99°$ **(b)** Given S (-3; 2) and O (2; - 4) determine the inclination of SO. $m_{SO} = \frac{y_2-y_1}{x_2-x_1}$ $m_{SO} = \frac{(-4)-(2}{(2)-(-3}$ $m_{SO} = \frac{(-4-2)}{(2+3)}$ $\boldsymbol{m_{SO} = -\frac{6}{5}}$ $\therefore \tan\theta = \boldsymbol{m_{SO}}$ To get θ , arc(shift) tan $-\frac{6}{5}$ $\therefore \quad \theta = 129{,}8°$
Equation of a line		
$y = mx + c$	Use when m and c is known.	Determine the equation of the line with gradient $\frac{8}{5}$ and y-intercept -3. Equation of the line is: $y = \frac{8}{5}x - 3$
$y - y_1 = m(x - x_1)$	Use when 1 point and a gradient is given.	Determine the equation of the line through $P(4;3)$ with gradient -1. $y - y_1 = m(x - x_1)$ $\therefore \quad y - 3 = -1(x - 4)$ $\therefore \quad y - 3 = -x + 4$
$\boldsymbol{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)}$	Use when 2 points are given.	Determine the equation of the line passing through two points $A(-3;-5)$ and $B(2;3)$ $\boldsymbol{y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1)}$ $y - (-5) = \frac{3-(-5)}{2-(-3)}(x - (-3))$ $y + 5 = \frac{3+5}{2+3}(x + 3)$ $y + 5 = \frac{8}{5}(x + 3)$ $y = \frac{8}{5}x + \frac{24}{5} - 5$ $y = \frac{8}{5}x - \frac{1}{5}$

Quadrilateral	Definition	Sides	Angles	Diagonals	Diagram
trapezium	A quadrilateral with one pair of opposite sides parallel	1 pr of opp sides =	Int ∠s sum to 360°	Cut each other	
kite	A quadrilateral with two pairs of adjacent sides equal	2 prs of adjacent sides =	• 1 pr opp ∠s = • Int ∠s sum to 360°	• Diagonals intersect ⊥ • 1 diagonal bisects the other • 1 diagonal bisects 2 opp int ∠s	
parallelogram	A quadrilateral with two pairs opposite sides parallel	• Prs of opp sides = • Prs of opp sides //	• Int ∠s sum to 360° • Prs of opp ∠s =	• Diagonals bisect each other • Diagonal bisects the Area	
rhombus	A parallelogram with all sides equal	• All sides = • Prs of sides //	• Int ∠s sum to 360° • Prs of opp ∠s =	• Diagonals bisect each other ⊥ • Diagonal bisects the opp int ∠s • Diagonal bisects the Area	
rectangle	A parallelogram with two pairs of parallel sides and a 90° angle.	• Prs of opp sides = • Prs of opp sides //	• All int ∠s = 90° • Int ∠s sum to 360°	• Diagonals bisect each other and are = • Diagonal bisects the Area	
square	A rectangle/ rhombus with all sides equal and a 90° angle.	• All sides = • Prs of sides //	• All int ∠s = 90° • Int ∠s sum to 360°	• Diagonals bisect each other ⊥ and are = • Diagonal bisects the int ∠s • Diagonal bisects the Area	

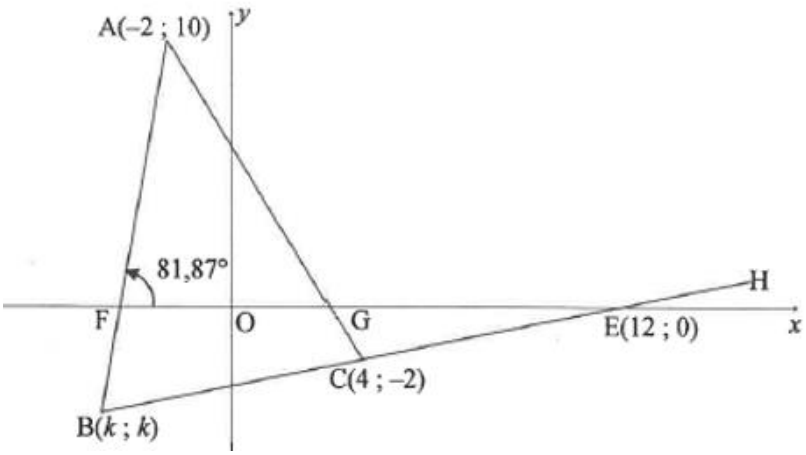
SESSION 3: Special Quadrilaterals

In Analytical Geometry questions can be asked which require you to know the properties of the various quadrilaterals. The properties of the various quadrilaterals you have learnt in grade 10, it is given in the table below. The table below captures the various properties you have to show a particular quadrilateral has for it to be a Parallelogram, Rectangle, Rhombus, Square, Trapezium or Kite.

Quadrilaterals	Used to prove whether a four-sided figure is the indicated quadrilateral that is Parallelogram, Rectangle, Rhombus, Square, Trapezium or Kite.
A. Parallelogram	• 1 pair of opposite sides are = and // • 2 pairs of opposite sides are = • 2 pairs of opposite sides are // • 2 pairs of opposite ∠'s are = • Diagonals bisect each other **any one**
B. Rectangle	• prove that the figure is a parallelogram + 1 angle = 90 + diagonals are = **+ any one**
C. Rhombus	• prove that the figure is a parallelogram + diagonals ⊥ + pair adjacent sides are = + diagonals bisect ∠'s **+ any one**
D. Square	• prove that the figure is a rhombus and one angle 90° **OR** • prove that the figure is a rectangle and diagonals are perpendicular **OR** • prove that the figure is a rectangle AND 1 pair adjacent sides are = **+ any one of these**
E. Trapezium	1 pair opposite sides are //
F. Kite	• pair of adjacent sides are = + other pair of adjacent sides = **OR** • diagonals ⊥ + one diagonal bisect the other diagonal **+ any one**

SESSION 3: EXAMPLES

1. In the diagram, A(-2 ; 10), B(k ; k) and C(4 ; -2) are the vertices of ΔABC. Line BC is produced to H and cuts the x-axis at E(12 ; 0). AB and AC intersect the x-axis at F and G respectively. The angle of inclination of line AB is 81.87°.



Determine the equation of the line :

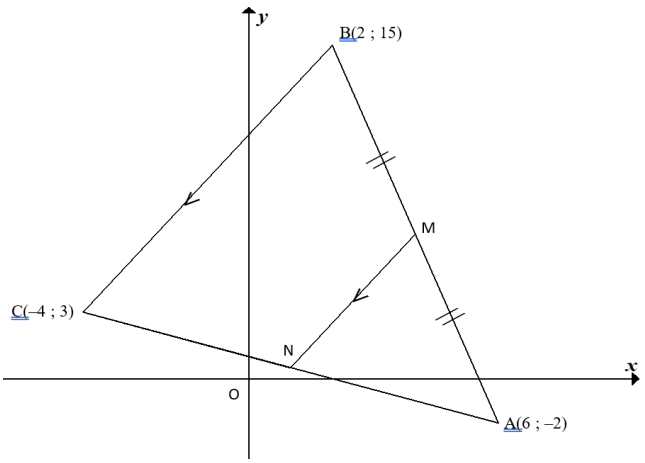
1.1 AF

1.2 BH

1.3 Passing through A, parallel to BH

1.4 Passing through C, perpendicular to BH

2. In the diagram, A(6 ; – 2), B(2 ; 15) and C(– 4 ; 3) are the vertices of ΔABC. M is the midpoint of AB. N is a point on CA such that MN || BC.



2.1 Determine the coordinates of M, the midpoint of AB.

2.2 Determine the gradient of line MN.

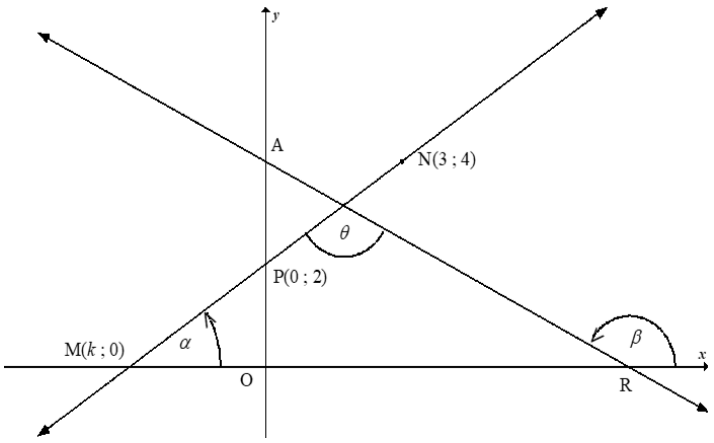
2.3 Hence, or otherwise, determine the equation of line MN.

2.4 Calculate, with reasons, the coordinates of point N.

2.5 If ABCD (in that order) is a parallelogram, determine the coordinates of point D.

SESSION 3: EXAMPLES continued

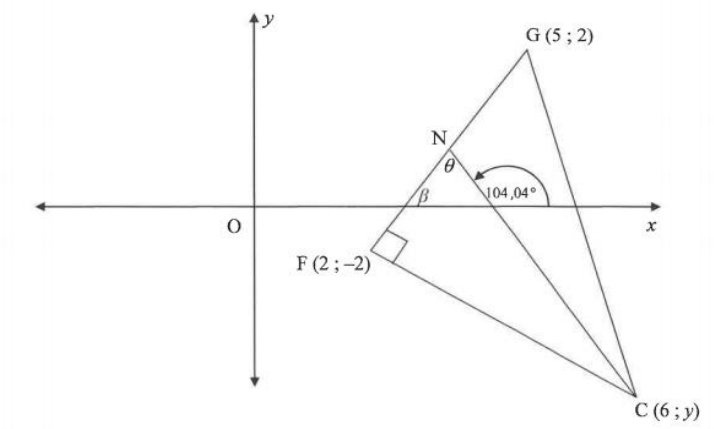
3. In the diagram, R and A are the x- and y-intercepts respectively of the straight line AR. The equation of AR is $y = -\frac{1}{2}x + 4$. Another straight line cuts the y-axis at P(0 ; 2) and passes through the points M(k ; 0) and N(3 ; 4). α and β are the angles of inclination of the lines MN and AR respectively.



3.1 If it is given that $k = -\frac{4}{3}$, determine the size of θ.

3.2 Determine the area of ΔMNR.

4. In the diagram, $F(2; -2)$, $G(5: 2)$ and $C(6; y)$ are the vertices of ΔFGC, $FG \perp FC$. N is a point on FG such that the inclination of NC is 104,04°.
The angle of inclination of FG is β and $\angle FNC = \theta$

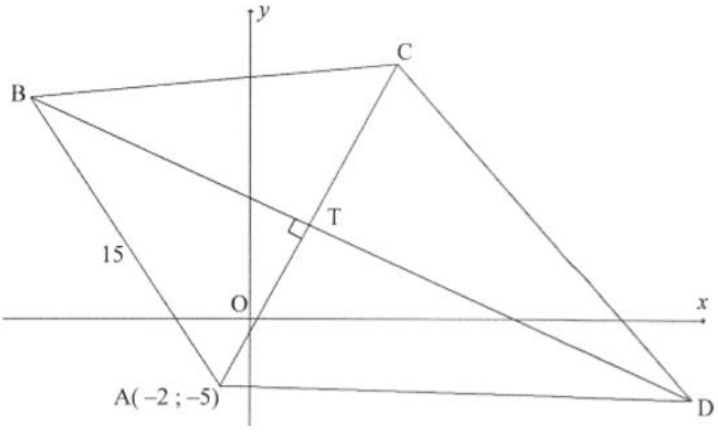


4.1 Determine the size of θ.

4.2 Calculate the value of y.

4.3 Determine the area of ΔFGC

5. $A(-2; -5), B, C$ *and* D are the vertices of Quadrilateral $ABCD$ such that diagonal AC is perpendicular to diagonal BD at T.
The equation of BTD is given by $2y + x = 18$ and $AB = 15$ units



5.1 Determine the gradient of line AC

5.2 Determine the equation of AC in the form $y = mx + c$

5.3 If the equation of AC is $y = 2x - 1$ calculate the coordinates of T.

5.4 If $ABCD$ is a kite with $AB = BC$

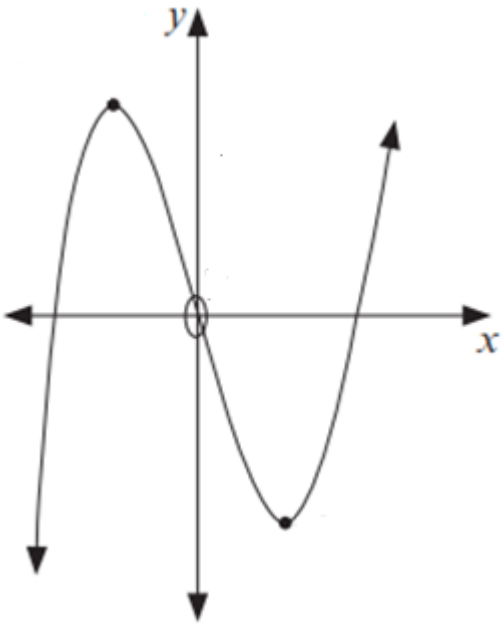
a) Determine the coordinates of C

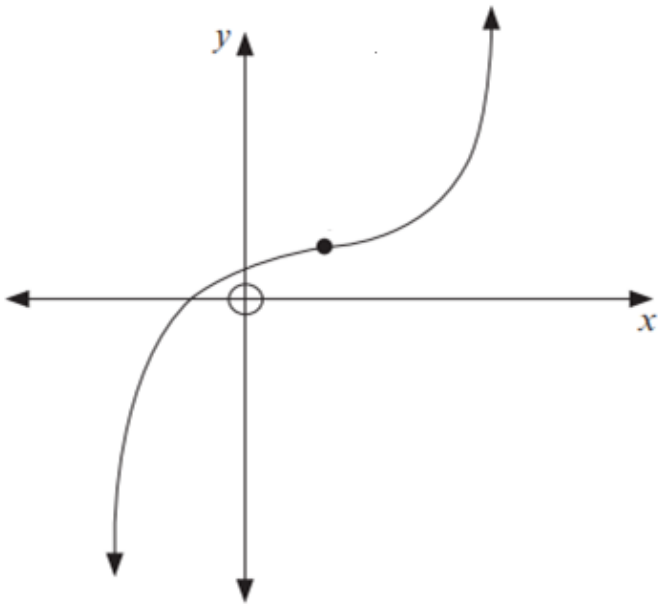
b) Calculate the length of BT

SESSION 5: CUBIC GRAPHS

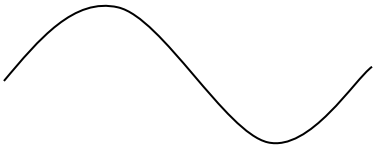
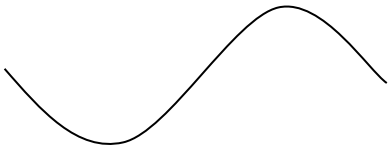
$$f(x) = ax^3 + bx^2 + cx + d$$

- Stationary points
- Local Maximum/Local Minimum
- Point of inflection
- Concave up/Concave down





SKETCHING THE GRAPH

If $a > 0$ 	**If** $a < 0$ 

y-INTERCEPT • Substitute $x = 0$ • Solve for y	**x-INTERCEPT** • Substitute $y = 0$ • Solve for x by factorising
TURNING POINT • Determine $f'(x)$ • $f'(x) = 0$ • Solve x • Substitute x into original formula	**POINT OF INFLECTION** • Determine $f''(x)$ • $f''(x) = 0$ • Solve for x • Substitute x into original formula

SESSION 5: CUBIC GRAPHS

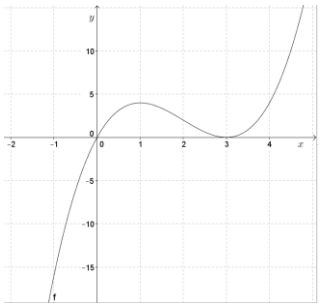
- DIFFERENT FORMS OF THE CUBIC FUNCTION
  1. $g(x) = ax^3 + bx^2 + cx + d$
  2. $k(x) = a(x - x_1)(x - x_2)(x - x_3)$
  3. $l(x) = a(x - x_1)^2(x - x_2)$
  4. $s(x) = a(x - x_1)(x^2 + mx + n)$

✓ Which of the above functions numbered 1 – 4 will you link with the sketches labelled A to F?

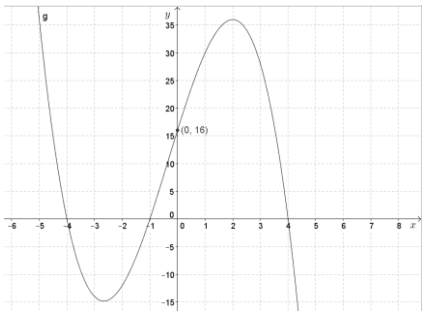
✓ What information of the cubic equation is given in each of the sketches?

✓ In each sketch will you be able to find the equation of the function? If not, what further information do you require in each case?

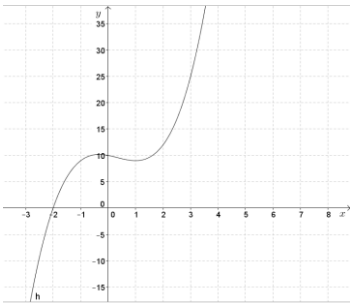
A



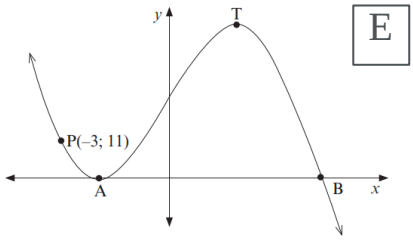
B



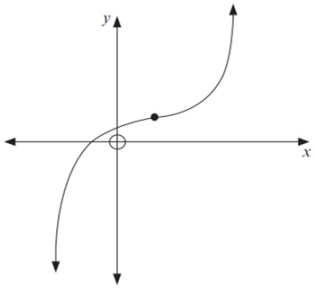
C



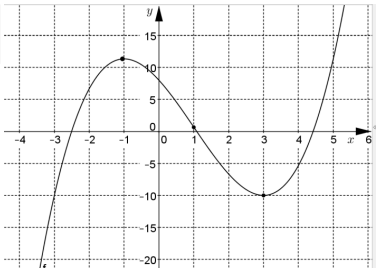
D



E



F



EXAMPLE 1:

In the given diagram, is the sketch of the function,

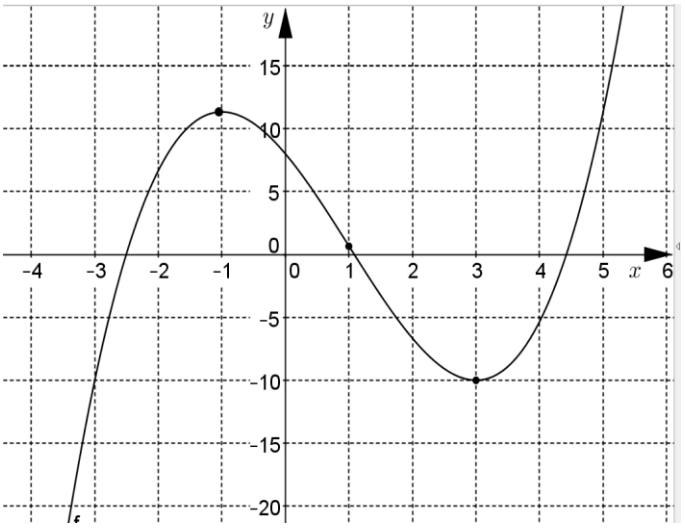
$f(x) = \frac{2}{3}x^3 - 2x^2 - 6x + 8$

Sketch the following on the given diagram :

(Show all x –intercepts and possible turning points)

2.1 $f'(x)$

2.2 $f''(x)$

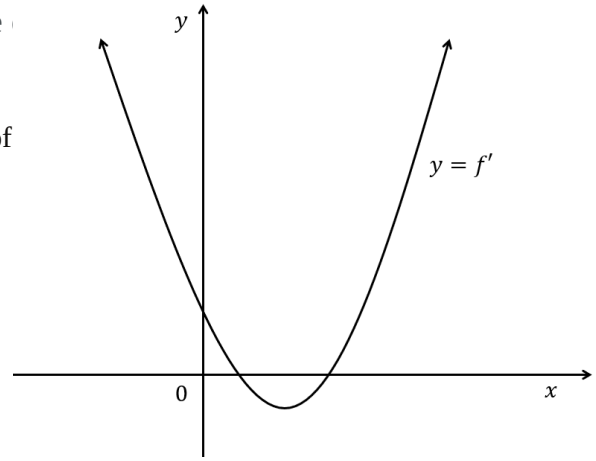


SESSION 5: CUBIC GRAPHS

EXAMPLE 2:

The graph of $y = ax^2 + bx + c$ represent the derivative

It is given that $f'(1) = 0$, $f'(3) = 0$ and $f'(0) = 6$

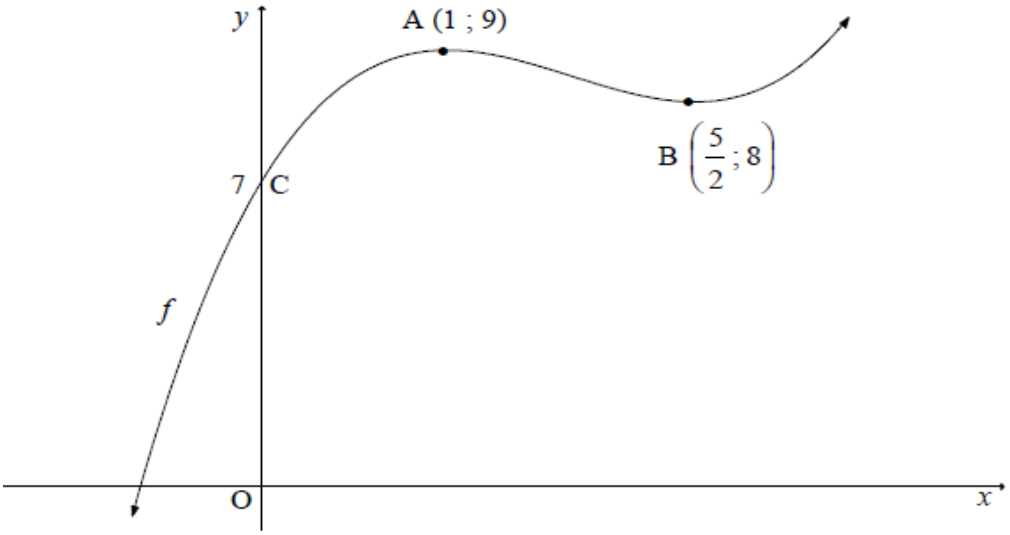


2.1 Write down the $x-$coordinates of the stationary points of

2.2 For which value(s) of x is f strictly decreasing?

Exercises:

1. A(1 ; 9) and B($\frac{5}{2}$; 8) are the turning points of the graph f below.
C(0 ; 7) is the y-intercept of f.



1.1 For which values of x is f decreasing? (2)

1.2 Write down the x-intercepts of f', the derivative of f (2)

1.3 For which values of x will f be concave up? (2)

1.4 Determine the value of k for which $y = f(x) + k$ will have THREE positive x-intercepts. (2)

SESSION 5: CUBIC GRAPHS

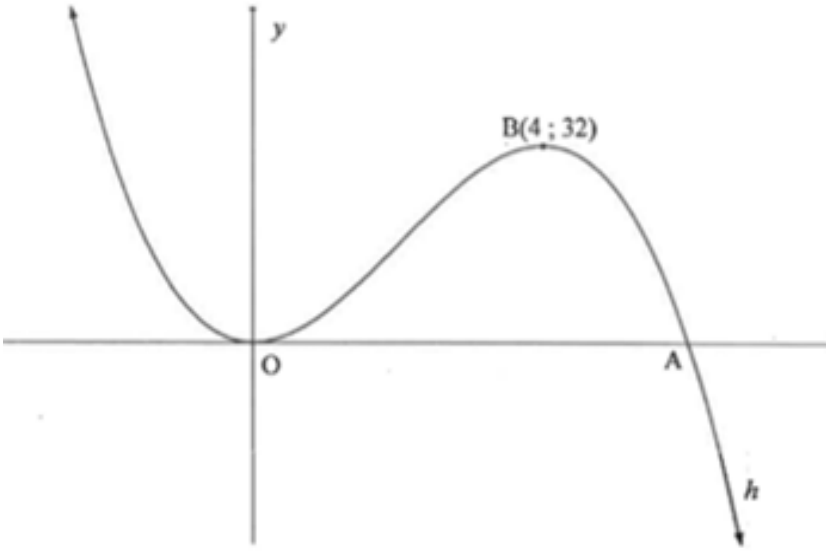
Exercises:

2. The graph of $h(x) = ax^3 + bx^2$ is drawn

The graph has turning points at the origin,

O(0 ; 0) and B(4 ; 32).

A is an x- intercept of h.



2.1 Show that $a = -1$ and $b = 6$ (5)

2.2 Calculate the coordinates of A. (3)

2.3 Write down the values of x for which h is:

2.3.1 Increasing (2)

2.3.2 Concave down (2)

3. Given $f(x) = x(x-3)^2$ with $f'(1) = f'(3) = 0$ and $f(1) = 4$.

3.1 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (4)

3.2 For which values of x will $y = -f(x)$ be concave down? (2)

3.3 Use your graph to answer the following questions:

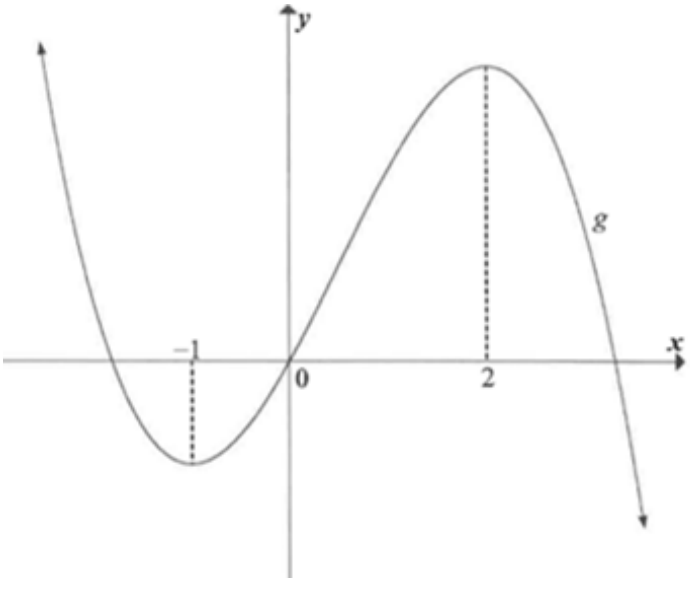
3.3.1 Determine the coordinates of the local maximum of h if ,

$h(x) = f(x-2) + 3$. (2)

3.3.2 Claire claims that $f'(2) = 1$.

Do you agree with Claire? Justify your answer. (2)

4. The graph of $g(x) = ax^3 + bx^2 + cx$, a cubic function having a $y-$ intercept of 0, is drawn. The x –coordinates of the turning points of g are -1 and 2.



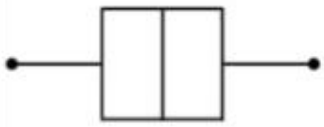
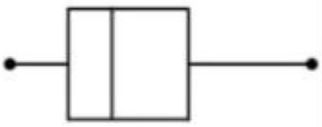
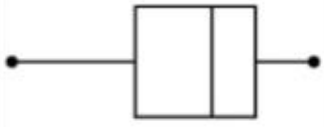
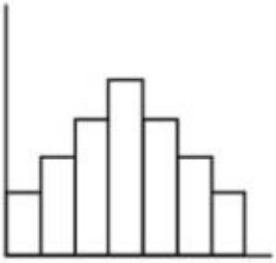
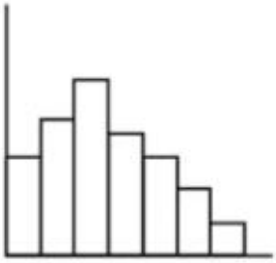
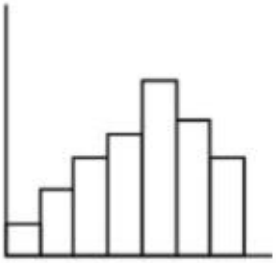
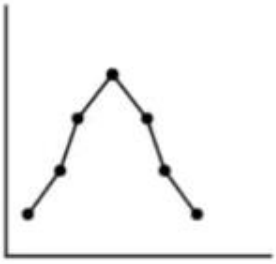
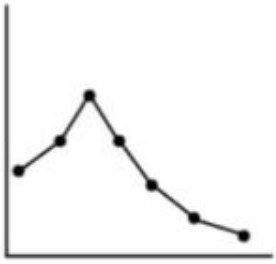
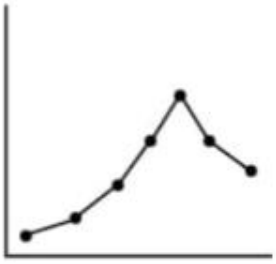
4.1 For which values of x will g increase? (2)

4.2 Write down the $x-$ coordinates of the point of inflection of g. (2)

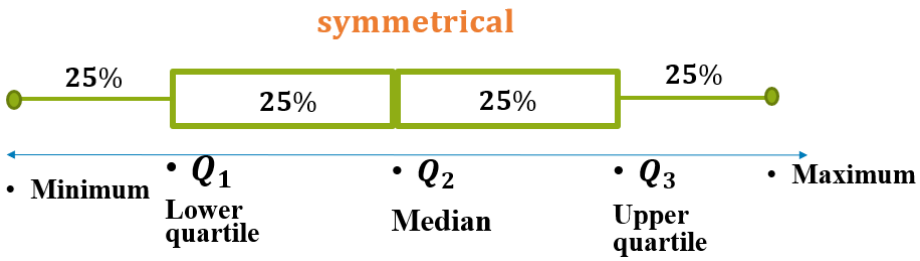
4.3 For which values of x will g be concave down? (2)

4.4 If $g'(x) = -6x^2 + 6x + 12$, determine the equation of g. (4)

SESSION 7: STATISTICS

INFORMATION	SYMMETRICAL	POSITIVELY SKEWED (Skewed to the right)	NEGATIVELY SKEWED (Skewed to the left)
Mean and Median	Mean = Median	Mean > Median	Mean < Median
Box-and-Whisker Diagram			
Histogram			
Frequency Polygon			
Distribution			

BOX AND WHISKER



Five number summary

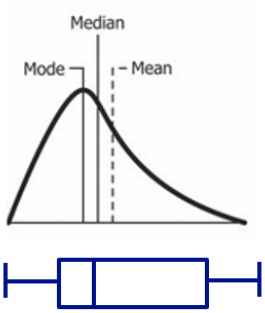
- **Minimum**
- $Q_{1(position)} = \frac{1}{4}(n+1)$
- $Q_{2(position)} = \frac{1}{2}(n+1)$
- $Q_{3(position)} = \frac{3}{4}(n+1)$
- **Maximum**

SESSION 7: STATISTICS - BOX and WHISKER & DISTRIBUTION

Distribution of data

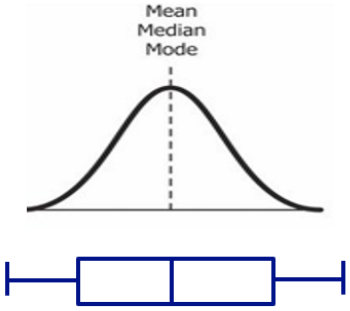
Positively skewed

- A high volume of data around the lower values of the data set.
- The higher values of the data set are more spread out.
- Mean > Median



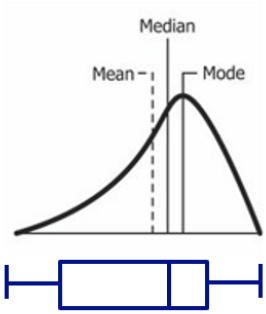
Symmetrical distribution

- A high volume of data situated around the mean.
- Data is symmetrical distributed around middle.
- Mean = Median.



Negatively skewed

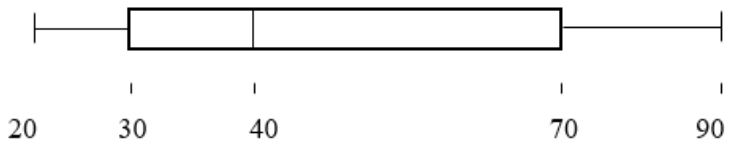
- A high volume of data around the higher values of the data set.
- The lower values of the data set are more spread out.
- Mean < Median.



- Skewness influences the mean. The more skew the data, the less the mean can be used for central tendency. It the data is skew to the left the mean is too low. If it is skew to the right the mean will be too high. The best measure of central tendency to use is the median, as this gives a better idea of what is happening to the central tendency of the data.

EXAMPLE: 1

The box and whisker diagram below show the marks (out of 90) obtained by a class of nine learners.



Answer the following questions.

1. Comment on the skewness of the data.
2. Why will the mean not be a good measure of central tendency?
3. Do you think the test was too easy or too difficult? Give a reason for your answer.

EXAMPLE: 2

The data below shows the number of people visiting a local clinic per day for 6 days.

12, 29, 13, 63, 12, 3

2.1 Draw a box and whisker diagram.

2.2 Use your calculator to determine the mean of the data

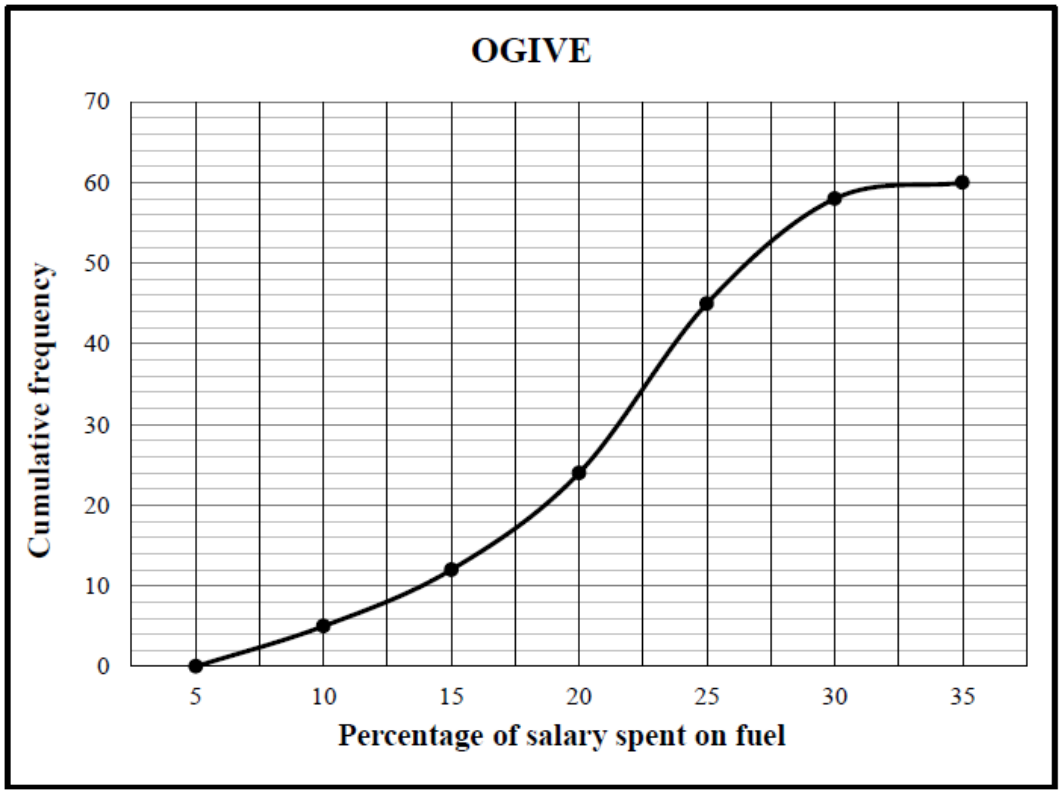
2.3 Comment on the skewness/ spread of the data

SESSION 7: STATISTICS - CUMULATIVE FREQUENCY GRAPH (OGIVE)

Cumulative frequency graph (ogive)	
What must I be able to do?	**What is important when you draw an ogive?**
• Interpret and make sense of an ogive. • Determine min, Q_1, Q_2, Q_3, maximum and percentiles from ogive. • Draw box & whisker plot from an ogive. • Be able to answer questions requiring you to read info from ogive. • Use a given ogive and complete a cumulative frequency table. • Use a cumulative frequency table to draw an ogive.	• **Complete the given frequency table:** ▪ **Add** a **cumulative frequency** column. ▪ Add a **coordinates** column. • Grid Paper: Add a heading, Mark and label axes. • Plot points. (Upper boundary of class ; cumulative frequency of class) • **GROUND** ogive at (lower boundary of first interval ; 0) • Connect the points with a smooth curve. You may not use a ruler. It is an S – shaped curve.

Interpretation of the OGIVE: Example 3

A company conducted research among all its employees on what percentage of their monthly salary was spent on fuel in a particular month. The data is represented in the ogive (cumulative frequency graph) below.



1. How many people are employed at this company? (1)
2. Write down the modal class of the data. (1)
3. How many employees spent more than 22,5% of their monthly salary on fuel? (2)
4. An employee spent R2 400 of his salary on fuel in that particular month. Determine the monthly salary of this employee if he spends 7% of his salary on fuel. (2)
5. Sketch the box & whisker plot from the given ogive.

SESSION 7: STATISTICS - CUMULATIVE FREQUENCY GRAPH (OGIVE)

SKETCHING AN OGIVE:

Example 4:

Each child in a group of four-year-old children was given the same puzzle to complete.

The time taken (in minutes) by each child to complete the puzzle is shown in the table below.

TIME TAKEN (t) (IN MINUTES)/ *TYD GENEEM (t) (IN MINUTE)*	NUMBER OF CHILDREN/ *GETAL KINDERS*
$2 < t \leq 6$	2
$6 < t \leq 10$	10
$10 < t \leq 14$	9
$14 < t \leq 18$	7
$18 < t \leq 22$	8
$22 < t \leq 26$	7
$26 < t \leq 30$	2

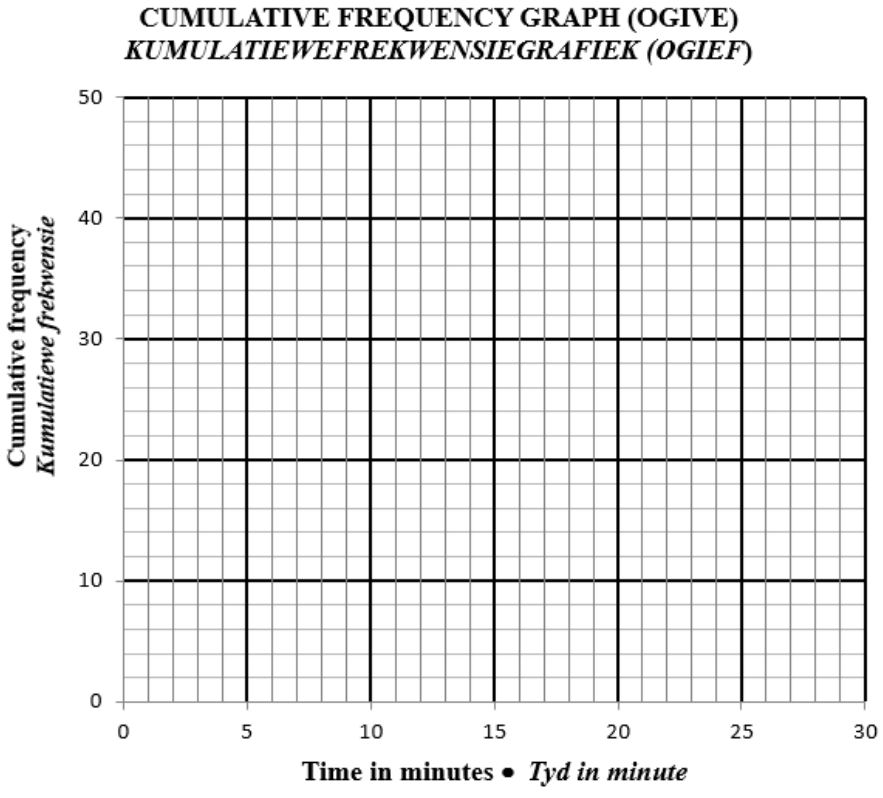
4.1 Draw the cumulative frequency graph(ogive) of the given data.

4.2 Use the graph to determine the median.

Solution:

4.1

Time in minutes (t) *Tyd in minute (t)*	Number of children *Getal kinders*	Cumulative frequency *Kumulatiewe frekwensie*
$2 < t \leq 6$	2	
$6 < t \leq 10$	10	
$10 < t \leq 14$	9	
$14 < t \leq 18$	7	
$18 < t \leq 22$	8	
$22 < t \leq 26$	7	
$26 < t \leq 30$	2	



SESSION 7: STATISTICS - STANDARD DEVIATION (SD) - σ

SD is a measure of dispersion using **all data** values. SD is how far data values **deviate** from the **mean.**
OR
the average **deviation** from the **mean**

Standard deviation(σ) is a quantity expressing by how much the members of a group differ from the mean value for the group. The bigger the value of the standard deviation the further away the data lies from the mean(x). This tells us that the mean is not the best measure of central tendency, and the median will be a better option. The smaller the value of the standard deviation, the closer the data is lying to the mean. Therefore, the mean is a reliable measurement of central tendency and can be used and trusted.

Determine the standard deviation - Calculator

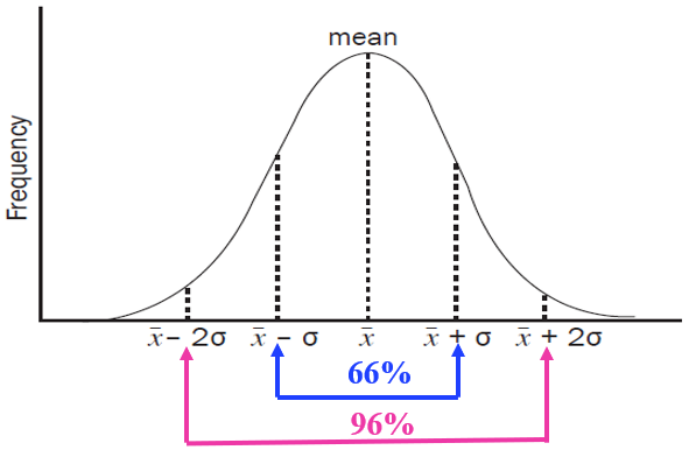
How to find standard deviation on Casio Calculator?

Press Mode (top right corner)
- then press 2 for STAT mode
- then press 1 corresponding to the 1-Var option.
- Now enter the data (= →enter & AC → save)
- Press SHIFT STAT (1)
- VAR (4)
- σx (3)

Interpretation of the standard deviation

The **Interquartile range** measures a spread around the **median**, so it has to do with the position of data and not their actual values.

The **Standard deviation** measures a spread around the **mean**, using the actual values of the data and not just their position.



Example 5:

The following sample of weekly wages of ten people was taken from a printing company.

R2 250; R2 250; R3 000; R3 300; R3 300

R3 600; R3 900; R4 350; R4 350; R5 250

5.1 Calculate the mean weekly wage.

5.2 Calculate the standard deviation of the weekly wage.

5.3 Determine the percentage of workers which lie within ONE standard deviation of the mean.

SESSION 7: STATISTICS EXAMINATION QUESTIONS

QUESTION 1

Mr Ngwane is the sales manager for a furniture shop. Every month his 15 staff members report on the number of customers who visited during the previous month.

The results were given as follows:

12 15 15 19 22 23 26 26 32 33 33 33 33 35 35

1.1 Determine the:

1.1.1 Median of the data (1)

1.1.2 Interquartile range (3)

1.1.3 Mean of the data (2)

1.1.4 Standard deviation of the data (2)

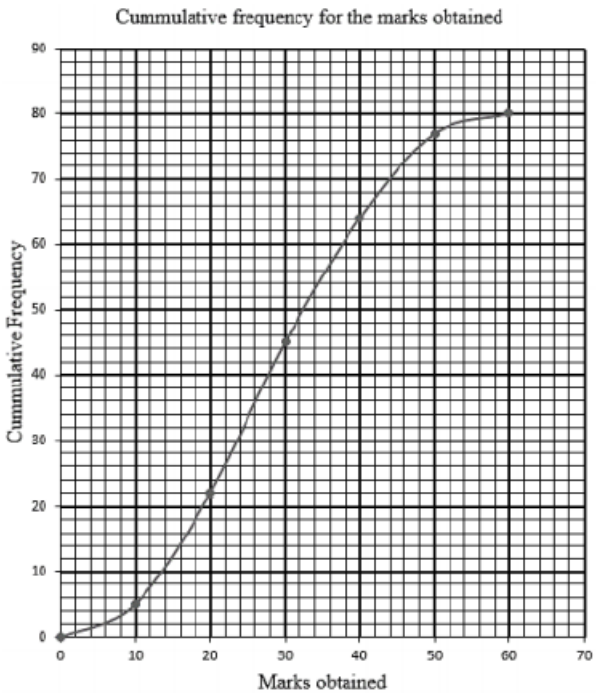
1.2 Determine the percentage of customers who visited the furniture shop that are outside one standard deviation of the mean (3)

[11]

QUESTION 2

A group of learners wrote a standardised English test that was scored out of 60.

The results were represented in a cumulative frequency graph below.



2.1 How many learners wrote the test? (1)

2.2 How many learners scored at least 20 out of 60? (2)

2.3 Using the graph, estimate the median test score. (2)

2.4 Complete the frequency table below (5)

Marks obtained	Frequency
$0<x\leq10$	
$10<x\leq20$	
$20<x\leq30$	
$30<x\leq40$	
$40<x\leq50$	
$50<x\leq60$	

2.5 Write down the modal group. (1)

[11]

QUESTION 3

A school held a sports day. One of the items on the program was an obstacle race. Teams of 10 parents and learners participated in the race. The table below shows the time taken, in minutes, by each member of a particular team to complete the race.

4	**12**	**13**	**16**	**17**	**18**	**20**	**22**	**22**	**25**

3.1 How long, in minutes, did it take for the fastest member of this team to complete the race? (1)

3.2 Determine the mean time taken by this team. (2)

3.3 Calculate the standard deviation for the data. (1)

3.4 How many members of the team completed the obstacle race outside of two standard deviations of the mean? (3)

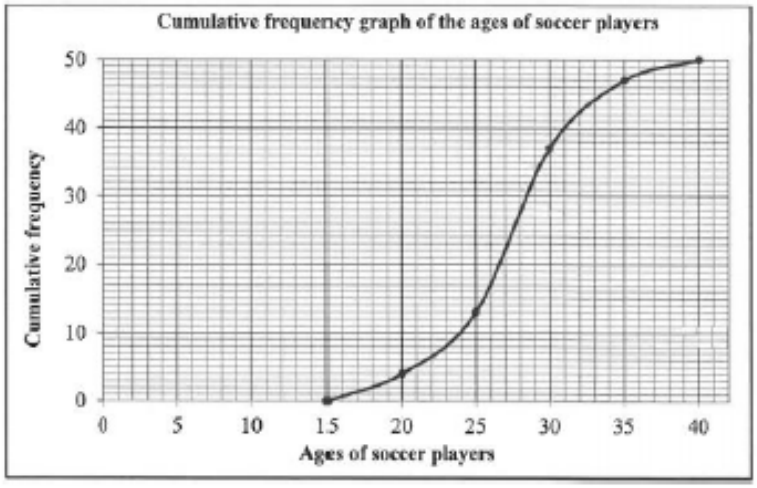
3.5 It took another team a total time of $\boldsymbol{x}+\mathbf{5}$ minutes to complete the race. Calculate the value of $\boldsymbol{x}$ if the overall mean of the two teams combined was 18 minutes. (3)

[10]

SESSION 7: STATISTICS EXAMINATION QUESTIONS

QUESTION 4

4.1 A survey was conducted of the ages of players at a soccer tournament. The results are shown in the cumulative frequency graph (ogive) below.



4.1.1 How many learners took part in the soccer tournament? (1)

4.1.2 Determine the number of players between the ages of 24 and 31 years old. (2)

4.1.3 Complete the frequency column of the table below in the answer book. (3)

CLASS INTERVAL	FREQUENCY	CUMULATIVE FREQUENCY
$15 \leq x < 20$		4
$20 \leq x < 25$		13
$25 \leq x < 30$		37
$30 \leq x < 35$		47
$35 \leq x < 40$		50

4.1 4 Draw a frequency polygon for the data(4)

4.2 Two grade 11 Mathematics classes have the same number of learners. The five-number summaries of the marks obtained by these classes for a test are shown below.

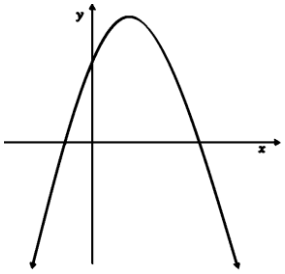
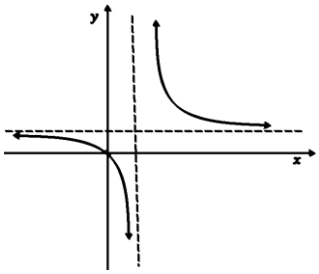
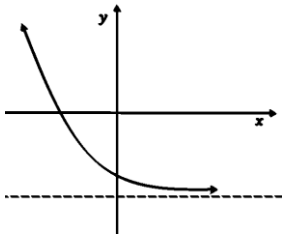
CLASS A (30; 48; 65; 82; 90)

CLASS B (50; 58; 65; 75; 90)

The parents of learners in CLASS A and CLASS B observe that both classes have the same median and the same maximum mark and therefore claim that there is no difference in the performance between these classes. Do you agree with this claim? Use at least two different arguments to justify your answer. (3)

[13]

FUNCTIONS: SUMMARY

GRAPH NAME	Parabola	Hyperbola	Exponential Graph
EQUATION	$y = ax^2 + bx + c$	$y = \frac{k}{x+p} + q$	$y = a^{x+p} + q$
WHAT YOU NEED to sketch the graph.	x −intercepts • $y = 0$	x −intercepts $y = 0$	x −intercepts $y = 0$
	y −intercepts • $x = 0$	y −intercepts $x = 0$	y −intercepts $x = 0$
	TURNING POINT Axes of Symmetry • $x = -\frac{b}{2a}$ • $f'(x) = 0$	ASYMPTOTES • $x = -p$ • $y = q$	ASYMPTOTE • $y = q$
EXAMPLE of the SKETCH of the graph			

Note:

The following are all forms of the parabola:

- $y = ax^2 + bx + c$
- $y = a(x + \mathrm{p})^2 + \mathrm{q}$
- $y = a(x - x_1)(x - x_2)$

Make sure :

- Given any equation of a function,
  - you can visualise what the shape of the graph is, that is whether it is a parabola, hyperbola or exponential. (this is by looking at the form of the equation and "a")
  - You are clear on the information you can write down by inspection.
  - You can sketch the function(Know what information is required in each of the different functions)
- Given a sketch:
  - You are able to by inspection identify what of the equation of the sketch is given.
  - You can identify what needs to be calculated in order to determine the equation of the function.

SESSION 9: FUNCTIONS - INSPECTION

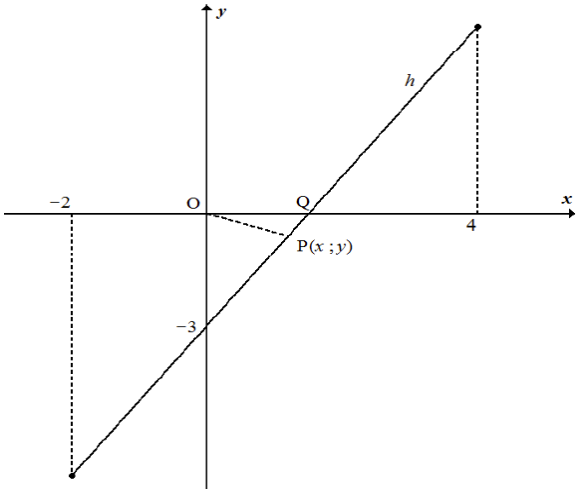
EQUATION	INFORMATION FROM INSPECTION				
$y = 2x^2 - 3x + 1$	$a = 2$, $a > 0$ & form of equation		$c = 1$	Parabola intersect/cuts the $y - axes$ at 1	Domain: $x \in \mathbb{R}$
$y = -(x-3)^2 + 2$	$a = -1$, $a < 0$ & form of equation		$TP(3;2)$	Parabola has a maximum value of 2 Axis of symmetry: $x = 3$	Domain: $x \in \mathbb{R}$ Range: $y \leq 3$
$y = -2x(x-5)$	$a = -1$, $a < 0$ & form of equation		Roots/ x −interce pts are: 0 and 5	Parabola intersect/cuts the $x - axis$ at 0 and 5 Axis of symmetry: $x = \frac{5}{2}$	Domain: $x \in \mathbb{R}$
$y = \frac{-1}{x-2} + 3$					
$y = 2^x - 1$					
$y = \frac{1}{x-2} + 3$					
$y = 2^{-x} - 1$					

1. How will each of the equations in the table above change if the sketch is shifted 2 units up along the y −axes?
2. For which of the above equations, can you by inspection write down the new equation if the graph is shifted horizontally to the right by 1 unit?

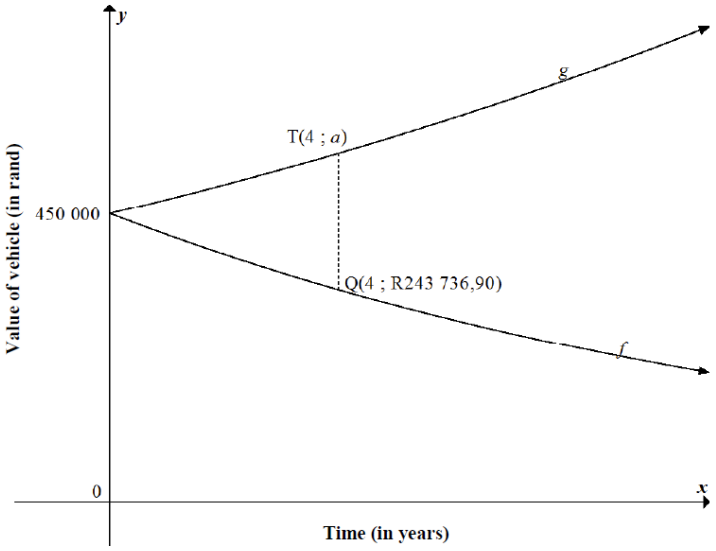
SESSION 9: FUNCTIONS – INSPECTION 1

In each of the following write down all the information from inspection:

1. Given: $h(x) = 2x - 3$ for $-2 \leq x \leq 4$. The x –intercept of h is Q

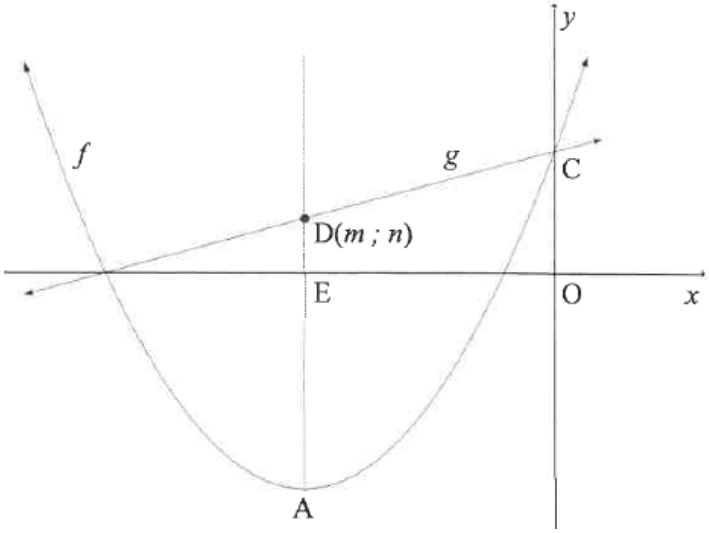


2. The graphs of f shows the book value of a vehicle x years after the time Joe bought it.
The graphs of g shows cost price of a similar new vehicle x years later.



3. The graphs of $f(x) = \frac{1}{2}(x+5)^2 - 8$ and $g(x) = \frac{1}{2}x + \frac{9}{2}$ are given in the sketch.

- A is the turning point of f.
- The axis of symmetry of f intersects the x-axis at E and the line g at $D(m\,;n)$.
- C is the y- intercept of f and g.



SESSION 9: FUNCTIONS – INSPECTION 2

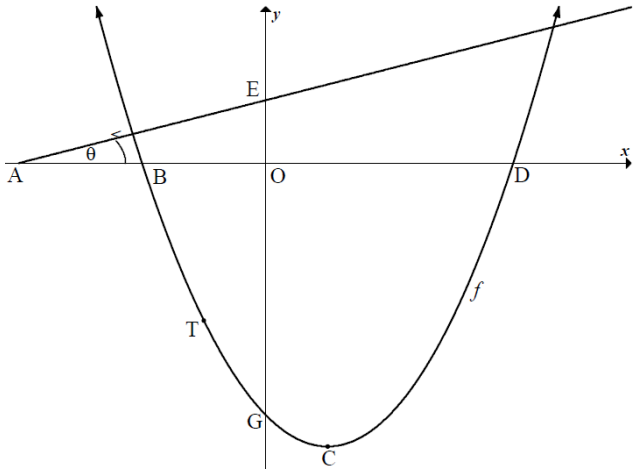
4.

The graph of $f(x) = (x + 4)(x - 6)$ is drawn.

The parabola cuts the x- axis at B and D and the y-axis at G. C is the turning point of f.

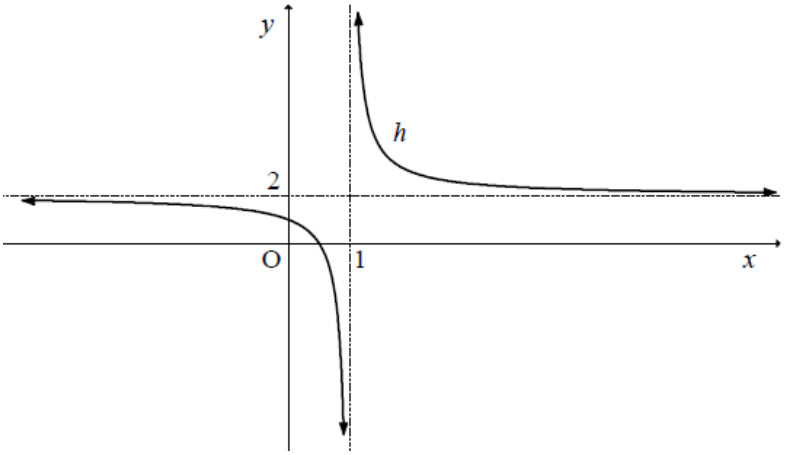
Line AE has an angle of inclination of θ and cuts the x-axis and y-axis at A and E respectively.

T is a point of f between B and G.



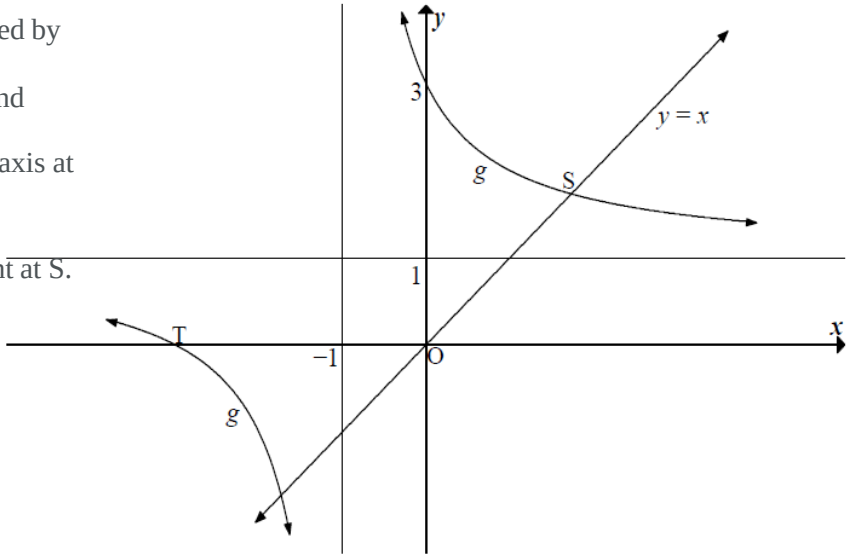
5.

The diagram shows the graph of $h(x) = \frac{1}{x+p} + q$. The asymptotes of h intersect at $(1\ ;\ 2)$.



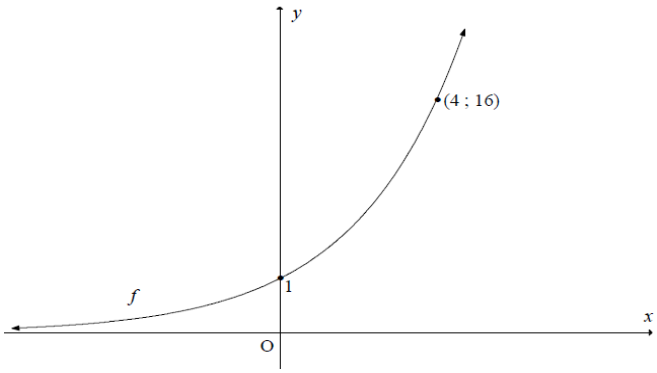
6.

The diagram shows the hyperbola g defined by $g(x) = \frac{2}{x+p} + q$ with asymptotes $y = 1$ and $x = -1$. The graph of g intersects the x-axis at T and the y-axis at $(0\ ;\ 3)$. The line $y = x$ intersects the hyperbola in the first quadrant at S.

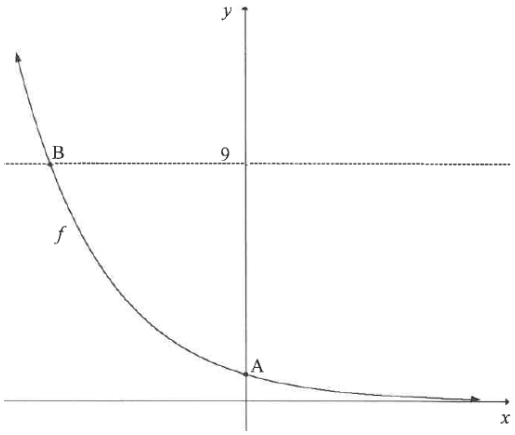


SESSION 9: FUNCTIONS – INSPECTION 3

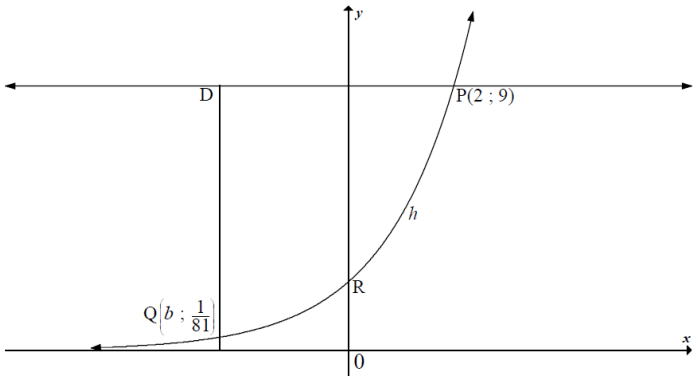
7.
Given is the graph of $f(x) = k^x$; $k > 0$.
The point $(4\,;16)$ lies on f.



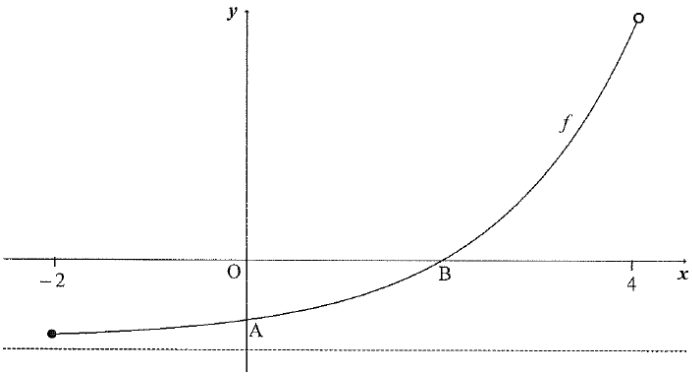
8.
The graph of $f(x) = 3^{-x}$ is sketched. A is the y-intercept of f. B is the point of intersection of f and the line $y = 9$.



9.
The graph of $h(x) = a^x$, $a > 0$. R is the y – intercept of h. The points $P(2\,;9)$ and $Q(b\,;\frac{1}{81})$ lie on h.



10.
The graph of $f(x) = 2^x - 4$ for $x \in [-2\,;4)$. A and B are respectively the y – and x-intercept of f.

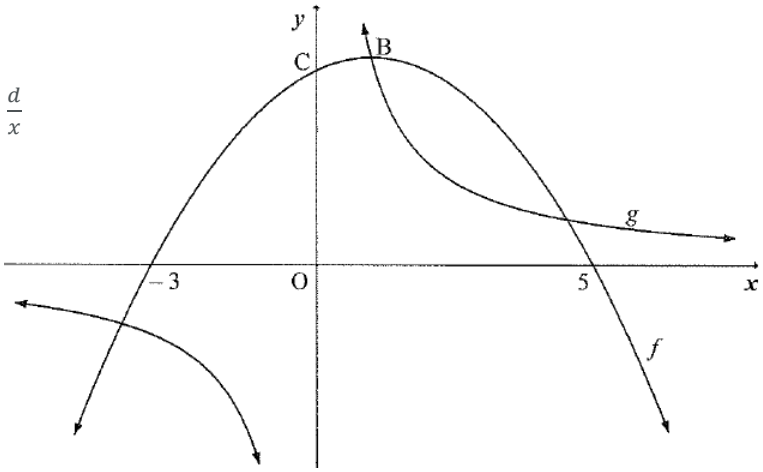


SESSION 9: FUNCTIONS – INSPECTION 2

11.

The graph of $f(x) = -\frac{1}{2}(x-1)^2 + 8$ and $g(x) = \frac{d}{x}$ are drawn.

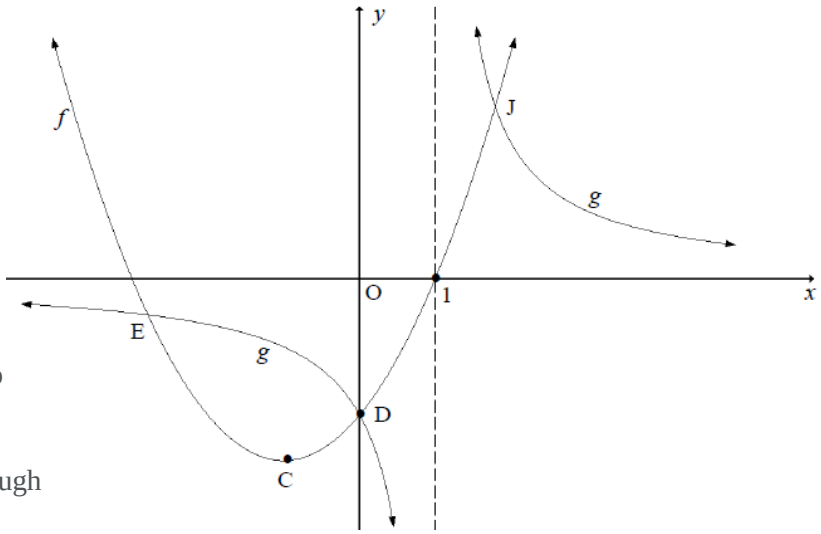
A point of intersection of f and g is B, the turning point of f. The graph of f has x- intercepts at (-3 ; 0) and (5 ; 0) and a y-intercept at C .



12.

The graphs of $f(x) = x^2 + bx - 3$ and $g(x) = \frac{a}{x+p}$ is given.

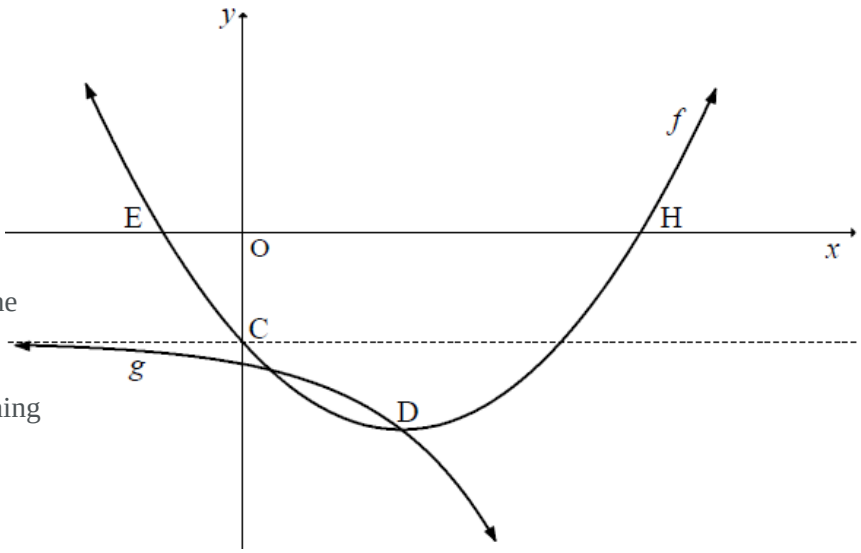
- f has a turning point at C and passes through the x- axis at $(1\ ;0)$
- D is the y-intercept of both f and g also intersect each other at E and J.
- The vertical asymptote of g passes through the x-intercept of f.



12.

The graphs of $f(x) = x^2 - 4x - 5$ and $g(x) = a.2^x + q$ are sketched.

- E and H are x-intercepts of f.
- C is the y-intercept of f and lies on the asymptote of g .
- The two graphs intersect at D, the turning point of f.



SESSION 9: THE PARABOLA

Sketching a parabola.

Equations in the TURNING POINT form: $y = a(x + p)^2 + q$

QUESTION 1:

Given: $f(x) = (x + 1)^2 - 4$

1.1 Write down the turning point of f.

1.2 Determine the x-intercepts of f.

1.3 Determine the coordinates of the y-intercept of f.

1.4 Draw the graph of f. Clearly show the turning point and intercepts with the axes.

1.5 Use your graph to answer the following questions:

1.5.1 Give the range of f.

1.5.2 For which values of x will

(a) $f(x) > 0$

1.5.3 Determine the equation of h where h is the graph after f moved 1 unit to the right.

1.5.4 Write down the turning point for g if g is the reflection of f in the x −axis.

Sketching a parabola.

Equations in the STANDARD FORM: $y = ax^2 + bx + c$

QUESTION 2:

Given: $f(x) = -2x^2 + 8x + 10$

2.1 Determine the coordinates of the turning point of f.

2.2 Determine the x-intercept of f.

2.3 Write down the coordinates of the y-intercept of f.

2.4 Draw the graph of f. Clearly show the turning point and intercepts with the axes.

2.5 Use your graph to answer the following questions:

2.5.1 Give the range of $f(x) - 3$.

2.5.2 For which values of x will

(a) $f(x) \leq 0$

2.5.3 Determine the equation of the axis of symmetry of $f(x - 1)$

2.5.4 Does the graph $-f(x)$ have a maximum or minimum value? Give this value.

SESSION 9: PARABOLA

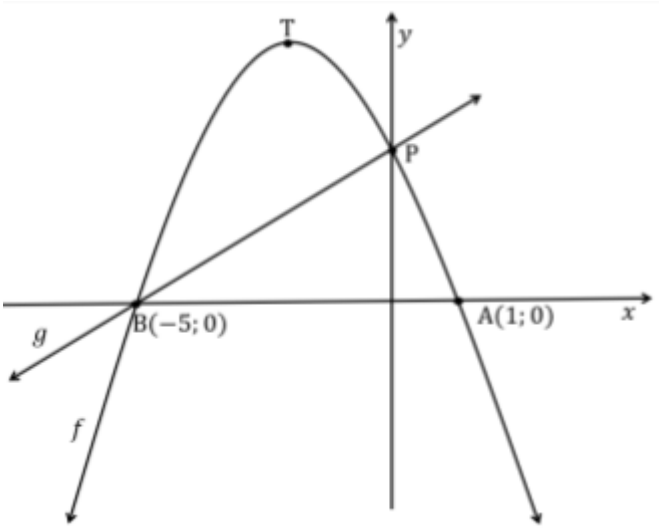
Determine the equation of a parabola.

x–INTERCEPTS GIVEN $y = a(x - r_1)(x + r_2)$

QUESTION 3:

The graph of $f(x) = ax^2 + bx + c$ and $k(x) = 2x + 10$ is drawn below.

- The parabola cuts the x-axis at A and B and the y-axis at P.
- B and P are also the intercepts of k.
- T is the turning point of f.



3.1 Write down:

3.1.1 the coordinates of P.

3.1.2 the equation of the axis of symmetry.

3.2 Determine the numerical values of a, b and c .

3.3 For which values of x is,

3.3.1 $f(x) \leq 0$

3.4 Give the range of f.

3.5 Determine the values of k for which $f(x) = k$ will have no real roots.

SESSION 9 : PARABOLA

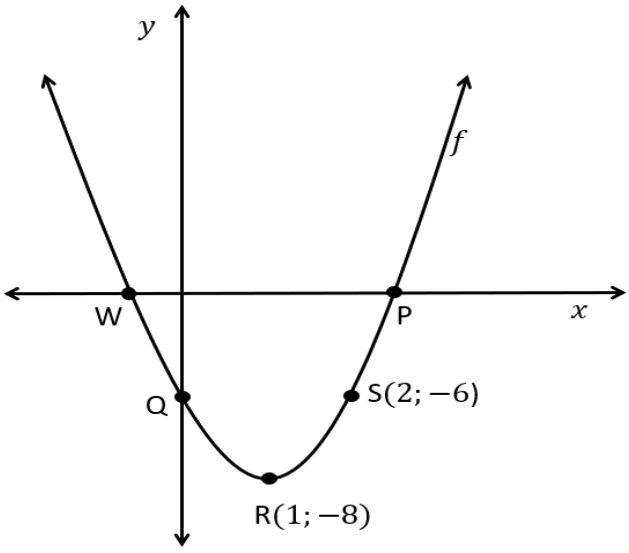
Determine the equation of a parabola.

TURNING POINT GIVEN $y = a(x + p)^2 + q$

QUESTION 4:

The graph of $f(x) = ax^2 + bx + c$ is drawn below. W, P and Q are the intercepts of f. R$(1\ ;\ -8)$ is the turning point of f. S$(2\ ;\ -6)$ is a point on f.

4.1 Write down the coordinates of Q if S and Q are reflections in the axis of symmetry.



4.2 Write down the range of $f(x) + 8$.

4.3 Determine the numerical values of a, b and c .

4.4 Determine the length of WP

4.5 For which values of x will

4.5.1 $f(x) \leq 0$

4.6 Write down the equation of the axis of symmetry for $f(x + 3)$

4.7 Determine the coordinates of the turning point for $f(-x)$.

4.8 Determine the values of k for which $f(x) = k$ will have two unequal, positive real roots.

SESSION 9 : HYPERBOLA

Sketching a hyperbola. $y = \frac{k}{x+p} + q$

QUESTION 5:

Given: $f(x) = \frac{6}{x-3} + 4$

5.1 Write down:

5.1.1 the equation of the horizontal asymptote of f.

5.1.2 the equation of the vertical asymptote of f.

5.1.3 the domain of f.

5.2 Determine:

5.2.1 the x-intercept of f.

5.2.2 the coordinates of the y-intercept of f.

5.3 Draw the graph of f. Clearly show ALL the asymptotes and intercepts with the axes.

5.4 Use your graph to answer the following questions:

5.4.1 Write down the range of f.

5.4.2 For which values of x will

(a) $f(x) \geq 0$

(a) $f(x) < 0$

5.5 Determine the equation of h where h is the graph after f moved 1 unit to the right.

5.6 Determine the equation(s) of the axis of symmetry of f.

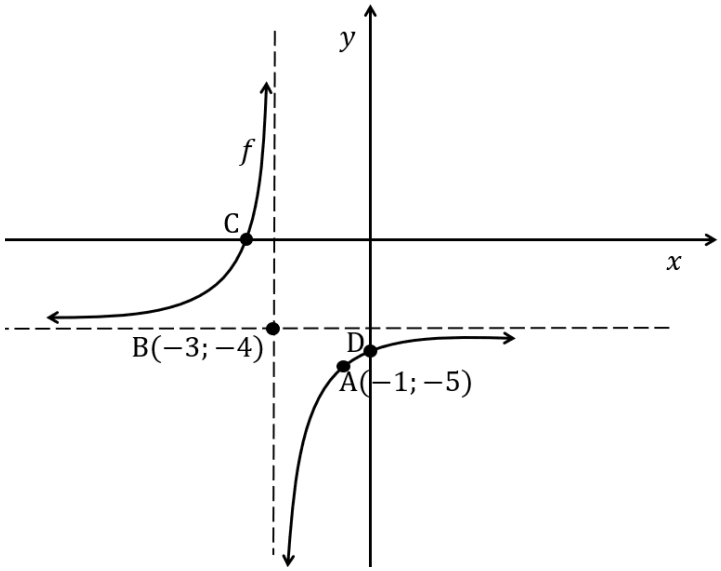
SESSION 9 : HYPERBOLA FUNCTION

Determine the equation of a hyperbola. $$y = \frac{k}{x+p} + q$$

QUESTION 6:

In the diagram is the graph of: $f(x) = \frac{k}{x+p} + q$. C and D are the intercepts of f.

The asymptotes of f intercept at B$(-3;-4)$. A $(-1;-5)$ is a point on f.



6.1 Write down:

6.1.1 the equations of the asymptotes of f.

6.1.2 the range of f.

6.1.3 the domain of $f(x+3)$.

6.2 Determine:

6.2.1 the numerical values of k, p and q.

6.2.2 the coordinates of the y-intercept of f.

6.2.3 the coordinates of the x-intercept of f.

6.2.4 the equation of the axis of symmetry of f that has a negative gradient.

6.3 For which values of x will:

6.3.1 $f(x) \geq 0$

6.3.2 $f(x) < 0$

6.4 Is the graph increasing or decreasing?

SESSION 9 : EXPONENTIAL FUNCTION

Sketching an exponential graph.

$$y = a.b^{x+p} + q$$

QUESTION 7:

Given: $f(x) = 2^{-x} - 4$

7.1 Write down the equation of the asymptote of f.

7.2 Determine:

7.2.1 the x-intercept of f.

7.2.2 the coordinates of the y-intercept of f.

7.3 Draw the graph of f. Clearly show ALL the asymptote(s) and intercepts with the axes.

7.4 Use your graph to answer the following questions:

7.4.1 Write down the range of f.

7.4.2 For which values of x will

(a) $f(x) \geq 0$

(a) $f(x) < 0$

7.5 Determine the equation of h where h is the graph after f moved 2 unit to the right and 1 unit up.

7.6 Is the graph increasing or decreasing?

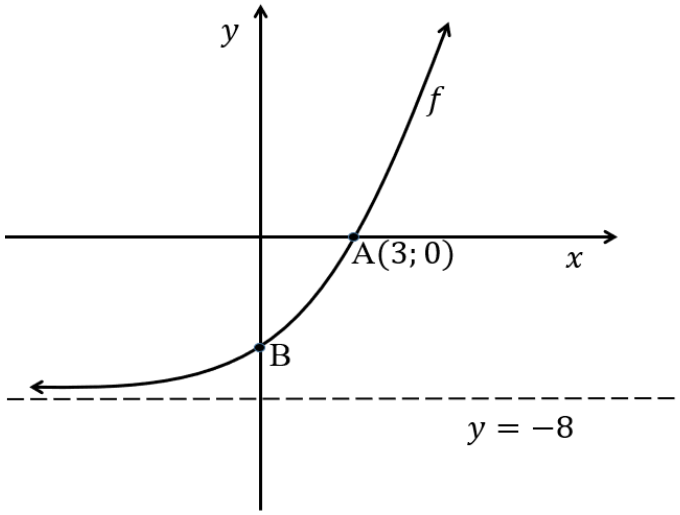
SESSION 5 : EXPONENTIAL FUNCTION

Determine the equation of an exponential graph. $y = a.b^{x+p} + q$

QUESTION 8:

In the diagram is the graph of: $f(x) = 2^{x+p} + q$. A(3;0) and B are the intercepts of f.

The asymptote of f is at $y = -8$.



8.1 Write down:

8.1.1 the range of f.

8.1.2 the range of $f(x) + 3$

8.2 Determine:

8.2.1 the numerical values of p and q.

8.2.2 the coordinates of B, the y-intercept of f.

8.3 For which values of x will:

8.3.1 $f(x) \geq 0$

8.3.2 $f(x) < 0$

8.4 Is the graph increasing or decreasing?

8.5 Describe the transformation from f to $h(x) = \left(\frac{1}{2}\right)^x$