Telematic Schools Project

2022 SUBJECT WORKBOOK
Grade 12

$f(x)=x^2$ MATHEMATICS

A joint initiative between the Western Cape Education Department and Stellenbosch University.

Western Cape Government
Education

Stellenbosch
UNIVERSITY
IYUNIVESITHI
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forward together
sonke siya phambili
saam vorentoe

BROADCAST SESSIONS

GRADE 12 **Geometry and Financial Mathematics**

Session	Date	Time	Topic
1	**28/02/2022**	**15h00-16h00**	**Geometry**
2	01/03/2022	15h00-16h00	Meetkunde
3	**25/08/2022**	**15h00-16h00**	**Financial Mathematics**
4	25/08/2022	16h00-17h00	Finansiële Wiskunde

INTRODUCTION AND TOPICS

Dear Grade 12 Learner

In 2022 there will be 4 live Grade 12 Telematic sessions, 2 of these sessions are in English and 2 in Afrikaans.

This workbook contain material for:
1. Grade 12 session on Geometry (28 February)
2. Grade 12 session on Financial Mathematics (25 August)

Please email, school@sun.ac.za with your name, surname, email and school name to register to obtain a username and password. You will then be able to access all previously recorded Telematic sessions and materials on other Topics from the Telematic Schools Project website, https://schools.sun.ac.za/ . Please note that accessing these recordings from this website is zero-rated.
At school you should receive a book called “Grade 12 Tips for Success”. In it you will have a breakdown of the weighting of the various Topics in Mathematics.
Ensure that you download a QR reader, this will enable you to scan the various QR codes.
The “MATHEMATICS EXAMINATION GUIDELINE”, can be downloaded by scanning the adjacent QR code or by clicking on, http://bit.ly/EXAM_GUIDE .
This examination guideline document contains important information regarding your end of year examination paper as well as the acceptable abbreviations to Geometry theorems which you are to use when answering Geometry questions.
At the start of each Telematic lesson, the presenters will provide you with a summary of the important
concepts and together with you will work though the exercises. You are encouraged to come prepared, have a pen and enough paper (ideally a hard cover exercise book) and your scientific calculator with you.
You are also encouraged to participate fully in each lesson by asking questions and working out the exercises, where you are asked to. Please, sms or e-mail your answers to the studio where you are asked to do so.
Remember:” Success is not an event, it is the result of regular and consistent hard work”.

Topics	Description
Geometry	**In this lesson we will discuss a strategy to study all Geometry Proofs you can be examined on at the end of the year. We will work through the different proofs.**
Financial Mathematics	This lesson will explain the Future Value and Present Value formula and it’s application.

SESSION 1: GEOMETRY TERMINOLOGY

TERM	EXPLANATION
theorem	A statement that has been proved based on previously established statements
radius	Straight line from the centre to the circumference of a circle or sphere. It is half of the circle's diameter
diameter	Straight line going through the centre of a circle connecting two points on the circumference
chord	Line segment connecting two points on a curve. When the chord passes through the centre of a circle it is called the diameter
circumference	The distance around the edge of a circle (or any curved shape). It is a the perimeter of a circle.
segment	The area bound by a chord and an arc.
arc	Part of the circumference of a circle
sector	The area bound by two radii and an arc
tangent	Line that intersects with a circle at only one point (the point of tangency)
Point of tangency	The point of intersection between a circle and its tangent line
exterior angle	The angle between any side of a shape, and a line extended from the next side.
Theorem of Pythagoras	In any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides
hypotenuse	The longest side in a right-angled triangle. It is opposite the right angle

SESSION 1 GEOMETRY TERMINOLOGY

TERM	EXPLANATION
complementary angles	Angles that add up to 90°
supplementary angles	Angles that add up to 180°
Vertically opposite angles	Non-adjacent opposite angles formed by intersecting lines
intersecting lines	Lines that cross each other
perpendicular lines	Lines that intersect each other at a right angle
corresponding angles	Angles that sit in the same position on each of the parallel lines in the position where the transversal crosses each line
alternate angles	Angles that lie on different parallel lines and on opposite sides of the transversal
co-interior angles	Angles that lie on different parallel lines and on the same side of the transversal
polygon	A closed 2D shape in which all the sides are made up of line segments. A polygon is given a name depending on the number of sides it has. A circle is not a polygon as although it is a closed 2D shape it is not made up of line segments
quadrilateral	A 4-sided closed shape (polygon)
cyclic quadrilateral	A quadrilateral whose vertices all lie on a single circle. This circle is called the circumcircle or circumscribed circle, and the vertices are said to be concyclic
diagonal	A straight line joining two opposite vertices (corners) of a straight sided shape. It goes from one corner to another but is not an edge
corollary	Theorem that follows on from another theorem

REVISION: GRADE 11 GEOMETRY THEOREMS AND CONVERSES

The proofs of the theorems marked with (**) must be studied as it could be examined.

1	Theorem**	The line drawn from the centre of a circle perpendicular to a chord bisects the chord; **(line from centre ⊥ to chord)**
	Converse**	The line from the centre of a circle to the midpoint of a chord is perpendicular to the chord. **(line from centre to midpt of chord)**
		The perpendicular bisector of a chord passes through the centre of the circle; **(perp bisector of chord)**
2	Theorem**	The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre); **(∠ at centre = 2 ×∠ at circumference)**
	Corollary	1. Angle in a semi-circle is 90^0 **(∠s in semi circle)** 2. Angles subtended by a chord of the circle, on the same side of the chord, are equal **(∠s in the same seg)** 3. Equal chords subtend equal angles at the circumference **(equal chords; equal ∠s)** 4. Equal chords subtend equal angles at the centre **(equal chords; equal ∠s)** 5. Equal chords in equal circles subtend equal angles at the circumference of the circles. **(equal circles; equal chords; equal ∠s)**
	Corollary Converse	1. If the angle subtended by a chord at the circumference of the circle is 90°, then the chord is a diameter. **(converse ∠s in semi circle)** 2. If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are concyclic.
3	Theorem**	The opposite angles of a cyclic quadrilateral are supplementary; **(opp ∠s of cyclic quad)**
	Converse	If the opposite angles of a quadrilateral are supplementary then the quadrilateral is a cyclic quadrilateral. **(converse opp ∠s of cyclic quad)**
	Corollary	The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle of the quadrilateral. **(ext ∠ of cyclic quad)**
	Corollary Converse	If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic. **(converse ext ∠ of cyclic quad)**
4	Theorem	The tangent to a circle is perpendicular to the radius/diameter of the circle at the point of contact. **(tan ⊥ radius)**
	Converse	If a line is drawn perpendicular to a radius/diameter at the point where the radius/diameter meets the circle, then the line is a tangent to the circle. **(line ⊥ radius)**
5	Theorem	Two tangents drawn to a circle from the same point outside the circle are equal in length. **(Tans from common pt OR Tans from same pt)**
6	Theorem**	The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment. **(tan chord theorem)**
	Converse	If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle. **(converse tan chord theorem OR ∠ between line and chord)**

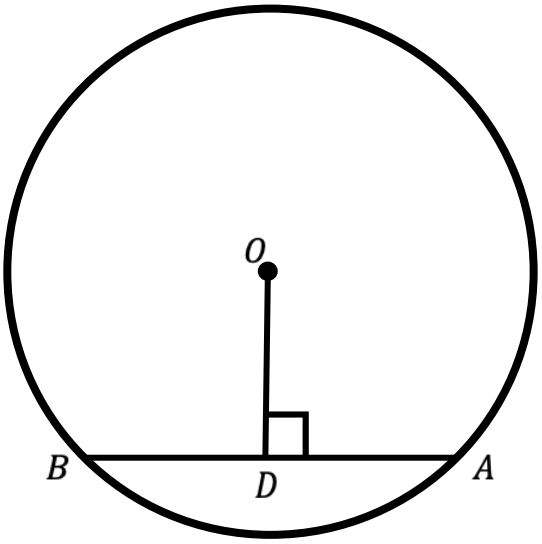
Below are Grade 12 Theorems, Converse Theorems and their Corollaries which you know. The proofs of the theorems marked with (**) must be studied because it could examined.

	Theorem**	A line drawn parallel to one side of a triangle divides the other two sides proportionally. **(line || one side of Δ OR prop theorem; name || lines)**
	Converse	If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side. **(line divides two sides of Δ in prop)**
	Theorem**	If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar) **(||| Δs OR equiangular Δs)**
	Converse	If the corresponding sides of two triangles are proportional, then the triangles are equiangular (and consequently the triangles are similar). Two variables are proportional if there is a constant ratio between them. **(Sides of Δ in prop)**

Question 1

Prove that the line drawn from the centre of a circle perpendicular to a chord bisects the chord;

(line from centre ⊥ to chord)



Required to prove:

Construction:

Proof:

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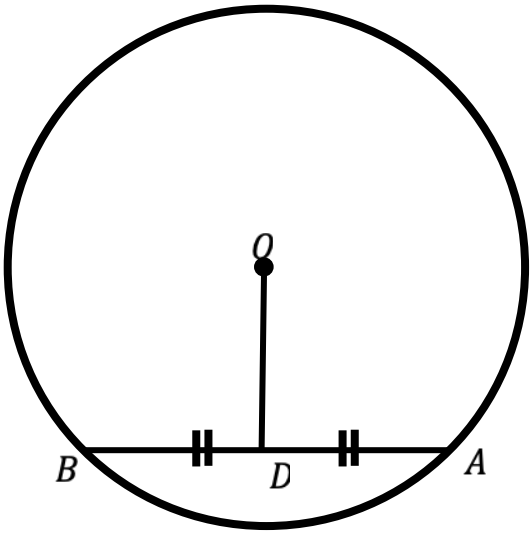
Question 2

Prove that the line from the centre of a circle to the midpoint of a chord is perpendicular to the chord.

(line from centre to midpt of chord)



Required to prove:

Construction:

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SESSION 1: GEOMETRY

SUMMARY

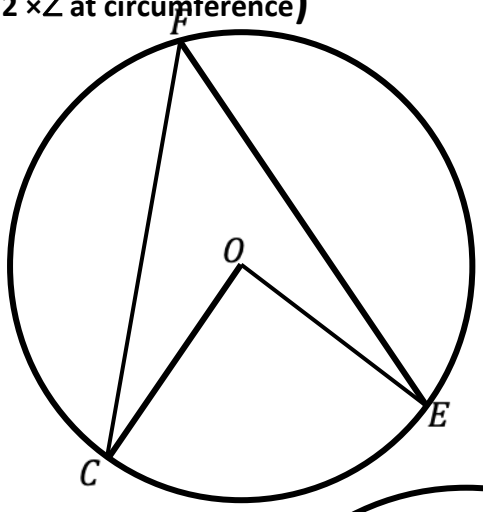
WHAT YOU SHOULD KNOW

1. Radii are equal.
2. Angles opposite equal sides are equal.
3. Exterior angle of a triangle is equal to the sum of the interior opposite angles..

Question 3

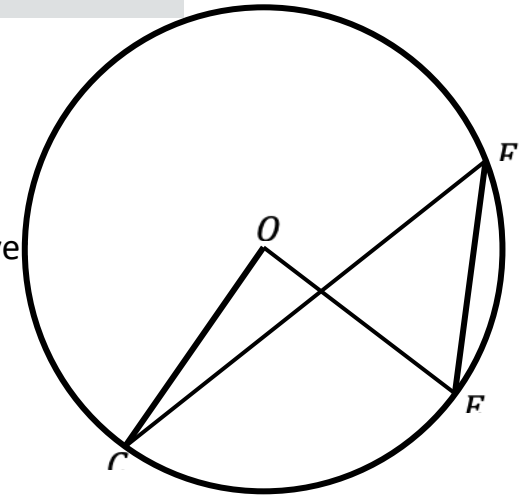
Prove the theorem, "The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre)".

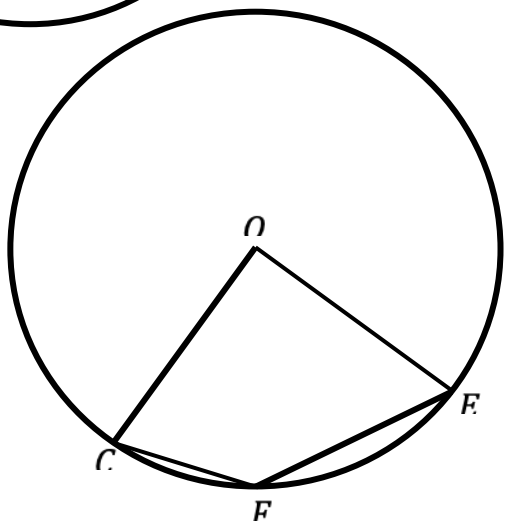
(∠ at centre = 2 ×∠ at circumference)



Required to prove

Construction:





Proof:

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SUMMARY

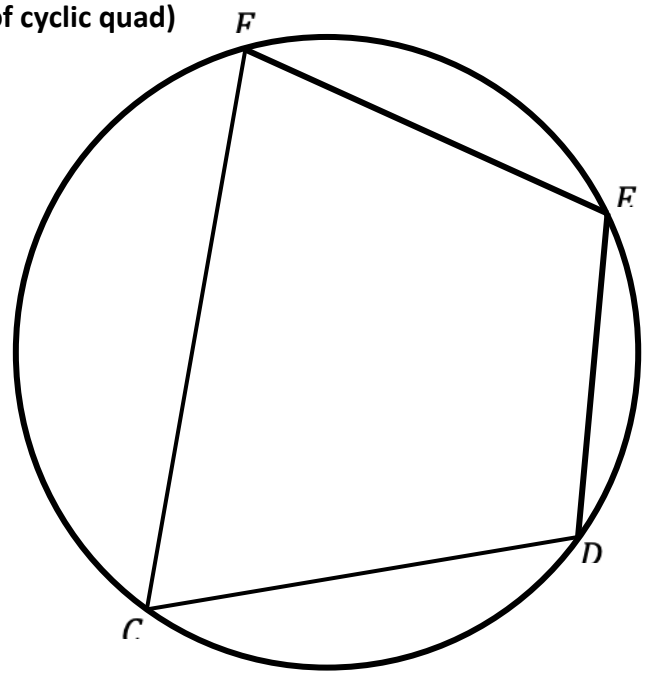
WHAT YOU SHOULD KNOW

1. The angle at the centre is twice angle at the circumference.
2. Angles around a point is 360°.

Question 4

Prove the theorem, "The opposite angles of a cyclic quadrilateral are supplementary".

(opp ∠s of cyclic quad)



Required to prove:

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SESSION 1: GEOMETRY

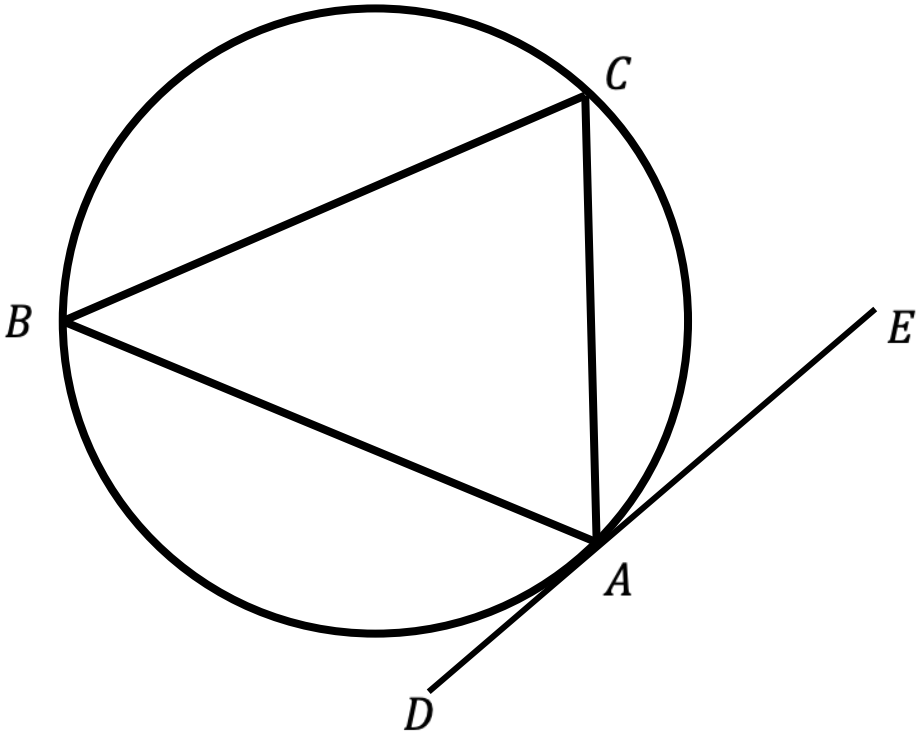
SUMMARY

WHAT YOU SHOULD KNOW

1. Radius is perpendicular to the tangent.
2. Angle in semi- circle is 90°.
3. Angles of a triangle is 180°.
4. Angles in the same segment are equal.

Question 5

Prove that the angle between the tangent and the chord is equal to the angle in the alternate segment. **(tan chord theorem)**



Required to prove:

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SESSION 1: GEOMETRY

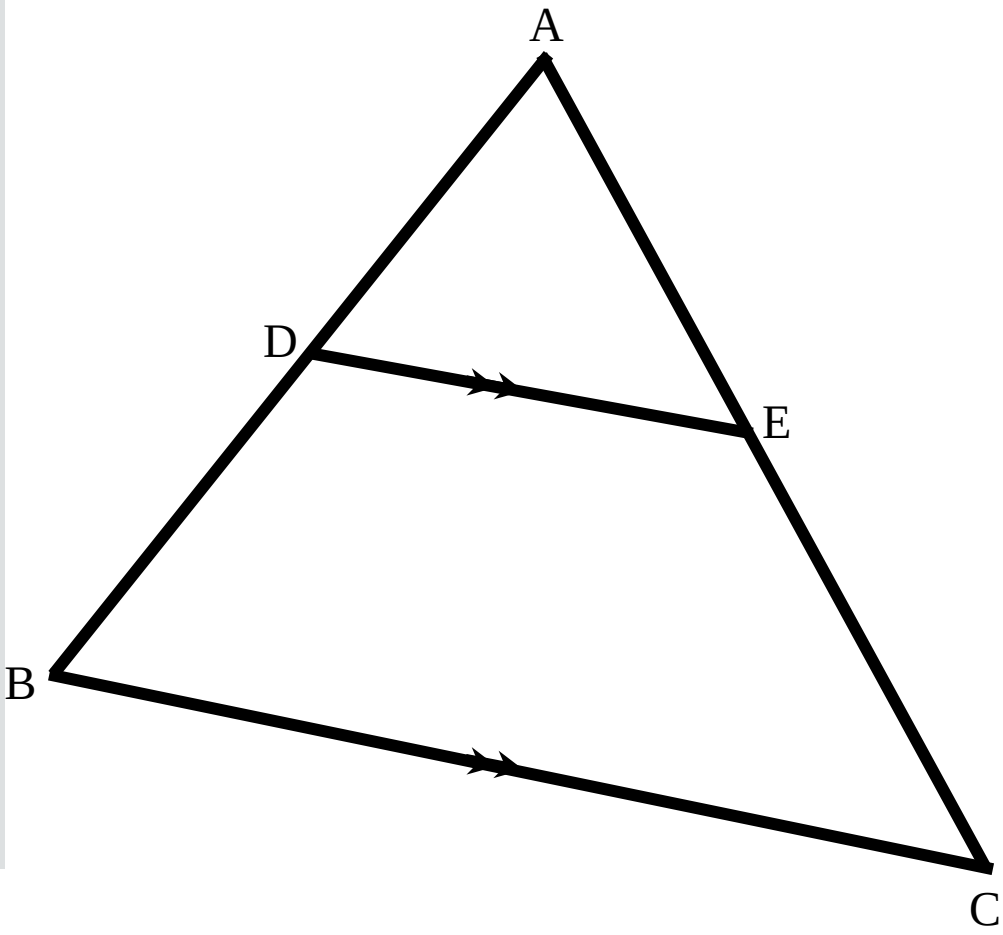
SUMMARY

WHAT YOU SHOULD KNOW

1. Formula for the Area of any triangle.
2. Triangles having the same vertex and whose base is on the same straight line have the same height.
3. Triangles on the same base between parallel lines have the same area.

Question 6

Use the diagram below to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally. (prop theorem)



Required to prove:

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SESSION 1: GEOMETRY

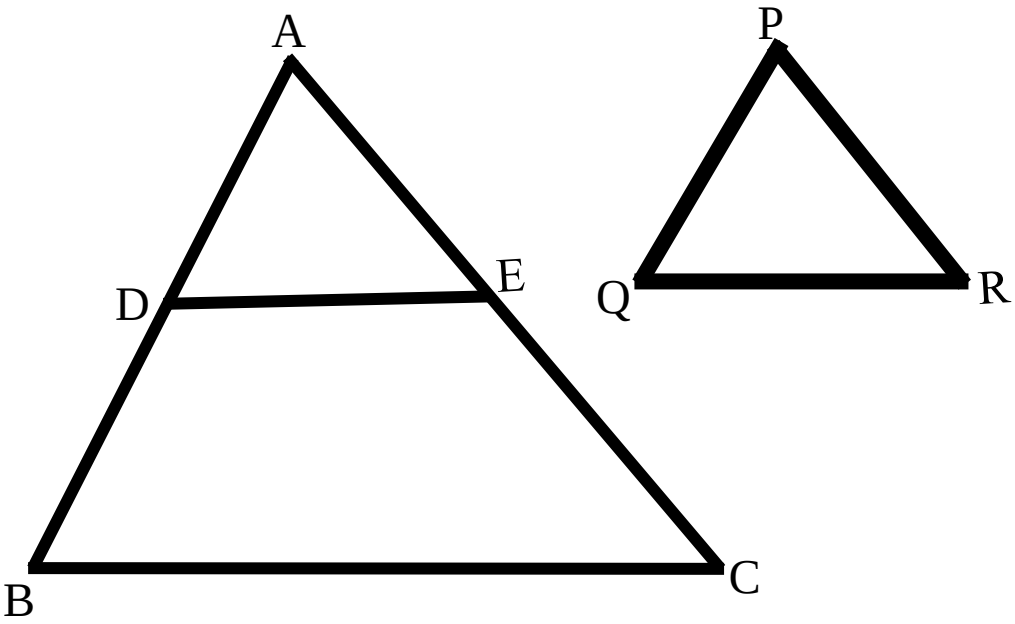
SUMMARY

WHAT YOU SHOULD KNOW

1. How to prove two triangle are congruent.
2. How to prove two lines are parallel.
3. A line joining two sides of a triangle parallel to a third divides the two sides proportionally.

Question 7

Use the diagram below to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally. (equiangular Δs)



Required to prove:

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SESSION 2: FINANCIAL MATHEMATICS TERMINOLOGY

Term	Explanation
principal amount	The initial or capital sum of money. Can represent an invested or borrowed amount of money.
profit	Money made when income exceeds (is more than) expenditure. You received more money than you spent.
loss	Money lost when expenditure exceeds income. (You spent more money than you made)
discount	Paying less than the normal price. A percentage of the original price is taken off.
budget	A plan to manage money.
interest	The cost of using someone else's money. It could be received for lending money or paid for borrowing money.
interest rate	The percentage of interest charged (loan) or received (investment). Rate is often quoted per annum.
simple interest	Interest on a loan, calculated on a yearly basis. Interest on a loan amount is charged taking only the principal amount into consideration.
compound interest	Interest is calculated not only on the principal amount but also includes any accumulated interest that has been added at certain time intervals.
instalment	A sum of money paid as one of several equal payments for something, spread over an agreed period of time.
exchange rate	The rate of one country's money against another country's money.
VAT	Value Added Tax is paid on goods or services in South Africa VAT is 15% of the price.
hire purchase	When an item is purchased but only a deposit is paid and then the item is paid off monthly over a specified period of time The interest charged is always simple interest. The goods bought are being hired until the payments are complete
deposit	A sum of money paid as a first instalment on an item with the understanding that the balance will be paid at a later stage.

SESSION 2: FINANCIAL MATHEMATICS TERMINOLOGY

Term	Explanation
inflation	A general increase in prices and fall in the purchasing value of money. Inflation is always calculated using compound interest.
fixed deposit	A single deposit invested for a certain period of time at a fixed interest rate.
depreciation	A reduction in the value of an asset over time, due in particular to wear and tear.
nominal interest	Nominal can also refer to the advertised or stated interest rate on a loan, without taking into account any fees or compounding of interest.
effective interest	The effective annual interest rate is the interest rate that is actually earned or paid on an investment, loan or other financial product due to the result of compounding over a given time period.
annuity	A series of payments at fixed intervals
Future value annuity	An annuity in which the aim of the payments is to save for the future. The future value of the annuity will be the total of the payments and interest.
present value annuity	An annuity in which the aim of the payments is to pay back a loan The present value of the annuity is the total amount borrowed.
deferred payment	Temporary postponement of the payment of an outstanding bill or debt, usually involving repayment by instalments.
sinking fund	Fund formed by periodically setting aside money for the gradual repayment to replace an asset that is past its useful lifetime.
pyramid scheme	A financial arrangement in which each paying participant recruits further participants, with returns being given to early participants using money contributed by later participants. It is illegal in many countries.

Simple Interest
$$A = P(1 + i \cdot n)$$

Compound Interest
$$A = P(1 + i)^n$$

Linear Depreciation
$$A = P(1 - i \cdot n)$$

Reducing Balance Method
$$A = P(1 - i)^n$$

Where

P is the principal (original sum of money invested or borrowed)

i is the interest (rate per period)

n is the number of years (no of periods)

A is the final amount

Nominal and effective interest rates: This formula is used to either calculate the nominal or effective interest rate, where m is the number of times per year interest is calculated.

$$i_{eff} = \left(\frac{1 + i^{(m)}}{m}\right)^m - 1$$

$i^{(m)}$ **- the nominal interest rate**

i_{eff} **- the effective interest rate**

Present Value

$$P_v = \frac{x[1 - (1 + i)^{-n}]}{i}$$

Future Value

$$F_v = \frac{x[(1 + i)^n - 1]}{i}$$

P_v- Present Value

F_V - Future Value

x – Regular payments

i – Interest rate per period

n – number of periods

SESSION 2: FINANCIAL MATHEMATICS

SUMMARY

WHAT YOU SHOULD KNOW

- First payment is at the **end** of the first period.
- Interest rate and payment periods are the same.
- Final payment at the **end** of the last period.

Question 1 (Adapted from NSC Nov 2020 Q6.1)

On 31 January 2020, Tshepo made the first of his monthly deposits of R1 000 into a savings account. He continues to make monthly deposits of R1 000 at the end of each month up until 31 January 2032. The interest rate was fixed at 7,5% p.a., compounded monthly.

1.1 What will the investment be worth immediately after the last deposit? (4)

1.2 If he makes no further payments but leaves the money in the account, how much money will be in the account on 31 January 2033? (2)

SESSION 2: FINANCIAL MATHEMATICS

SUMMARY

WHAT YOU SHOULD KNOW

- Loans are normally Present Value calculations.
- Interest paid=Total Payments made –Change in amount owed.

Question 2 (Adapted from NSC Nov 2020 Q6.3)

Mpho is granted a loan under the following conditions:

- The interest rate is 11,3% p.a., compounded monthly.
- The period of the loan is 6 years.
- The monthly repayment on the loan is R1 500.
- Her first repayment is made one month after the loan is granted.

2.1 Calculate the value of the loan. (3)

2.2 How much interest will Mpho pay in total over the first 5 years? (4)

SESSION 2: FINANCIAL MATHEMATICS

SUMMARY

WHAT YOU SHOULD KNOW

- Outstanding Balance of a Loan is the Present Value of the payments still to be made.
- Interest paid=Total Payments made –Change in amount owed.

Question 3 (Adapted from NSC Nov 2021 Q8.3)

Thabo wanted to save 8450 000 as a deposit to buy a house on 30 June 2018.

3.1 He paid in a fixed amount of money at the end of every month into an account earning interest of 8,35% p.a., compounded monthly. His first payment was made on 31 July 2013 and his 60th deposit on 30 June 2018. Calculate the amount he paid in monthly. (3)

SESSION 2: FINANCIAL MATHEMATICS

SUMMARY

WHAT YOU SHOULD KNOW

- Outstanding Balance of a Loan is the Present Value of the payments still to be made.
- Interest paid=Total Payments made –Change in amount owed.

Question 3 (Adapted from NSC Nov 2021 Q8.3)

Thabo wanted to save 8450 000 as a deposit to buy a house on 30 June 2018.

3.2 Thabo bought a house costing R1 500 000 and used his savings as the deposit. He obtained a home loan for the balance of the purchase price at an interest of 12% p.a., compounded monthly over 25 years. He made his first monthly instalment of R11 058,85 towards the loan on 31 July 2018.

(a) What will the balance outstanding on the loan be on 30 June 2039, 21 years after the loan was granted? (3)

(a) Calculate the interest Thabo will have paid over the first 21 years of the loan.

SESSION 2: FINANCIAL MATHEMATICS

SUMMARY

WHAT YOU SHOULD KNOW

- Increasing the repayments will reduce the loan term significantly.
- Interest paid=Total Payments made –Change in amount owed.

Question 4 (Adapted from NSC Nov 2019 Q6.2)

Nine years ago, a bank granted Mandy a home loan of R525 000. This loan was to be repaid over 20 years at an interest rate of 10% p.a., compounded monthly. Mandy's monthly repayments commenced exactly one month after the loan was granted.

4.1 Mandy decided to make monthly repayments of R6 000 instead of the required R5 066,36. How many payments will she make to settle the loan? (4)

4.2 After making monthly repayments of R6 000 for nine years, Mandy required money to fund her daughter's university fees. She approached the bank for another loan. Instead, the bank advised Mandy that the extra amount repaid every month could be regarded as an investment and that she could withdraw this full amount to fund her daughter's studies. Calculate the maximum amount that Mandy may withdraw from the loan account. (5)

SESSION 2: FINANCIAL MATHEMATICS

SUMMARY

WHAT YOU SHOULD KNOW

- Rates of depreciation are normally yearly effective rates.
- Use log when you determine the number of years, n.

Question 5 (Adapted from NSC Nov 2020 Q6.2)

Jim bought a new car for R250 000. The value of the car depreciated at a rate of 22% p.a. annually according to the reducing-balance method. After how many years will its book value be R92 537,64? (3)

SESSION 2: FINANCIAL MATHEMATICS

SUMMARY

WHAT YOU SHOULD KNOW

- Reducing balance method will not reach zero. It is similar to an exponential function with $0 < b < 1$.
- Book value is what the tractor is worth at a specific time.
- Use $A = P(1 + i)^n$ for single payments.
- First calculate the number of quarters and then the years.

Question 6 (Adapted from NSC Nov 2021 Q8.1)

A farmer bought a tractor for R980 000. The value of the tractor depreciates annually at a rate of 9,2% p.a. on the reducing-balance method. Calculate the book value of the tractor after 7 years. (3)

Question 7 (Adapted from NSC Nov 2021 Q8.2)

How many years will it take for an amount of R75 000 to accrue to R116 253,50 in an account earning interest of 6,8% p.a., compounded quarterly? (4)