Telematic Schools Project

2022 SUBJECT WORKBOOK
Grade 11

$f(x)=x^2$ **MATHEMATICS**

A joint initiative between the Western Cape Education Department and Stellenbosch University.

Western Cape Government
Education

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forward together
sonke siya phambili
saam vorentoe

BROADCAST SESSIONS

GRADE 11

Function

Session	Date	Time	Topic
1	**13/10/2022**	**15h00-16h00**	**Parabola & line**
2	17/10/2022	15h00-16h00	Parabool en Reguitlyn
3	**02/11/2022**	**15h00-16h00**	**Hyperbola and Exponential**
4	03/11/2022	15h00-16h00	Hiperbool en Eksponensiële

INTRODUCTION

Dear Grade 11 Learner

In 2022 there will be 4 Telematic sessions for grade 11 learners. Two sessions will be in English and two sessions will be in Afrikaans. This workbook provides you with material for these sessions. Please make sure that you bring this workbook along to each and every Telematic session. The first session will be on the Parabola and Straight line, and the second session on Hyperbola and Exponential function.

Please note all these grade 11 functions are vital and will be assessed in grade 12 as well. This content is approximately 25 marks of your grade 12 Mathematics Paper 1. So I really appeal to you to make every effort to understand these functions now in grade 11. You are indirectly preparing for the grade 12 examination as well.

Please email, school@sun.ac.za with your name, surname, email and school name to register to obtain a username and password. You will then be able to access all previously recorded Telematic sessions and materials on other Topics from the Telematic Schools Project website, https://schools.sun.ac.za/ . Please note that accessing these recordings from this website is zero-rated.

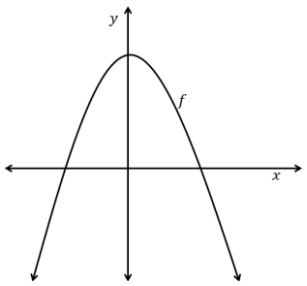
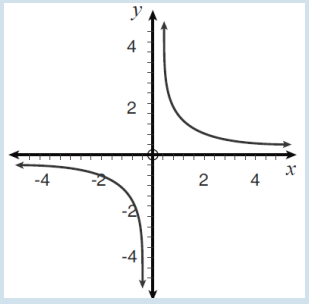
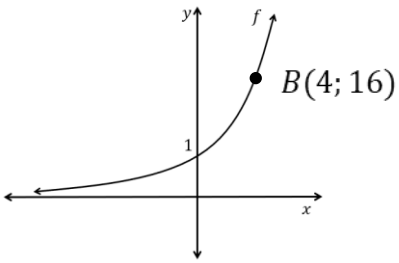
You are encouraged to come prepared, have a pen and enough paper (ideally a hard cover exercise book) and your scientific calculator with you.

You are also encouraged to participate fully in each lesson by asking questions and working out the exercises, and where you are asked to do so, sms or e-mail your answers to the studio.

Remember:" Success is not an event, it is the result of regular and consistent hard work".

GOODLUCK, Wishing you all the success you deserve!

TERMINOLOGY: FUNCTIONS

Term	Definition
function	**A relation for which each element of the domain (x) corresponds to exactly one element of the range (y). For every x-value there is only one possible y-value. To test for a function on a graph, use the 'horizontal line test' – run a ruler from left to right. If your ruler only ever touches the graph in one spot, it is a function.**
parabola	
hyperbola	
exponential	
increasing function	**A function that is going 'uphill' when looking at it from left to right. If a function has a turning point, this is the point at which a function could change from increasing to decreasing.**
decreasing function	**A function that is going 'downhill' when looking at it from left to right. If a function has a turning point, this is the point at which a function could change from decreasing to increasing.**

TERMINOLOGY

Term	Definition
horizontal shift	A translation of the graph either to the left or the right
vertical shift	A translation of the graph either up or down
average gradient	The gradient between two points on a curved graph.
domain	All the possible *x*-values that are covered by the function/graph. (To find the domain always study the graph from left to right)
range	All the possible *y*-values that are covered by the function/graph. (To find the range always study the graph from bottom to top)
***x*-intercept**	Where a function crosses (intercepts) the *x*-axis
***y*-intercept**	Where a function crosses (intercepts) the *y*-axis
axis of symmetry	A line that cuts the function exactly in half. A parabola has a vertical line of symmetry ($x = \ldots$) and a hyperbola has 2 axes of symmetry (which form a cross → *X* . These are in the form $y = mx + c$) An exponential graph does not have an axis of symmetry
asymptote	A straight line that a curved graph gets closer and closer to but never touches it.
turning point	The point at which a function (parabola, sin or cos graph) changes from increasing to decreasing OR from decreasing to increasing
maximum	Only involves functions that have a turning point. The maximum is the highest the graph can be and is always the *y*-value of the turning point
minimum	Only involves functions that have a turning point. The minimum is the lowest the graph can be and is always the *y*-value of the turning point.

PROPERTIES RELATING TO ALL FUNCTIONS

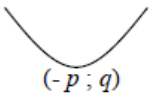
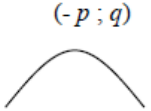
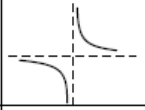
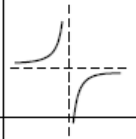
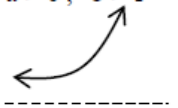
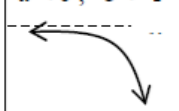
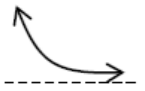
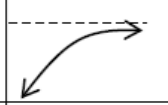
x intercept: Point on the x-axis where $y = 0$ (solve for x when $y = 0$) y-intercept: Point on the y-axis where $x = 0$ (substitute $x = 0$)

Domain: The set of all x-values that make the function true (usually $x \in R$, unless there is a vertical asymptote)

Range: The set of all y-values that make the function true.

TRANSFORMATIONS IN FUNCTIONS

$g(x) = f(-x)$	Reflection of f about the y-axis	
$g(x) = -f(x)$	Reflection of f about the x-axis	
$g(x) = f(x) + q$	Translation of f up or down q units	$q > 0$ UP , $q < 0$ DOWN
$g(x) = f(x + p)$	Translation of f left or right p units	$p > 0$ LEFT , $p < 0$ RIGHT
$g(x) = f(ax)$	Changes steepness in a graph (non-trigonometric)	

	Straight Line				Parabola		Hyperbola		Exponential	
Equation	$y = ax + q$				$y = ax^2 + bx + c$ or $y = a(x + p)^2 + q$		$y = \frac{a}{x+p} + q$		$y = a.b^x + q$	
Shape	$a > 0$ 	$a < 0$	$a = 0$ $\therefore y = q$ 	a undefined $\therefore x = \ldots$	$a > 0$  $(-p\,;q)$	$a < 0$ $(-p\,;q)$ 	$a > 0$ 	$a < 0$ 	$a > 0$; $b > 1$ 	$a < 0$; $b > 1$ 
					(Axis of Symmetry, Min Value) (A/S; Min V)	(Axis of Symmetry, Max Value) (A/S; Max V)			$a > 0$; $0 < b < 1$ 	$a < 0$; $0 < b < 1$ 
Domain	$x \in R$	$x \in R$	$x \in R$		$x \in R$	$x \in R$	$x \in R$- $\{-p\}$	$x \in R$- $\{-p\}$	$x \in R$	$x \in R$
Range	$y \in R$	$y \in R$		$y \in R$	$y \in [q\,;\infty)$	$y \in (-\infty\,;q]$	$y \in R$- $\{q\}$	$y \in R$- $\{q\}$	$y \in (q\,;\infty)$	$y \in (-\infty\,;q)$
Other Important points	a = gradient = $\frac{y_2 - y_1}{x_2 - x_1}$ q: y-value of the y-intercept				**Turning Point (-p ; q)** **To calculate the turning if** $y = ax^2 + bx + c$ For the x-coordinate, (A/S) $x = -\frac{b}{2a}$; this is equation of A/S For y-coordinate: substitute the calculated x-value into the equation		**Asymptotes:** $y = q$ $x = -p$ **Lines of symmetry:** $y = (x + p) + q$ $y = -[(x + p) + q]$		**Asymptotes:** $y = q$	

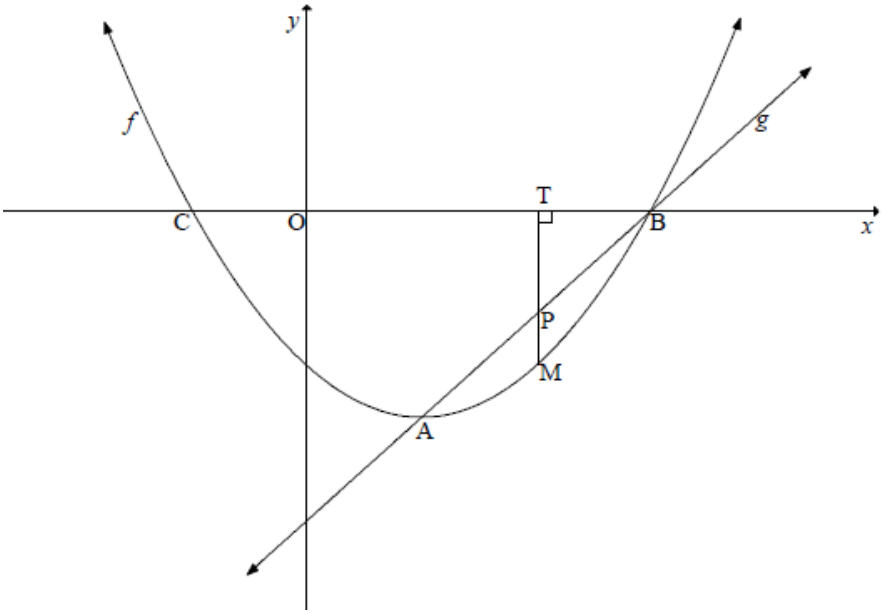
SUMMARY

WHAT YOU SHOULD KNOW

- All the terminology on page 4 and 5.
- Understand what the axis of symmetry is.
- Product of binomials.
- Understand turning point.
- How to determine equation of a straight line.

QUESTION 1

The graph of $f(x) = x^2 + bx + c$ and the straight line g are sketched below. A and B are the points of intersection of f and g. A is also the turning point of f. The graph of f intersects the x-axis at B(3 ; 0) and C. The axis of symmetry of f is $x = 1$.



1.1 Write down the coordinates of C.

1.2 Determine the equation of f in the form $y = ax^2 + bx + c$.

1.3 Determine the equation of g in the form $y = mx + c$.

1.4 Determine the Turning point of f .

1.5 Write down the range of f.

1.6 For which value(s) of x is f increasing?

1.7 For which value(s) of h will $f(x) = h$, have two positive roots.

1.8 If $k(x) = f(x + 1) - 2$, determine the equation of k , in the form, $k(x) = a(x + p)^2 + q$.

1.9 For which values of x will: $\frac{f(x)}{g(x)} > 0$

1.10 Determine the value(s) for x, for which $\frac{4}{f(x)}$ is a maximum.

SESSION 1: PARABOLA AND STRAIGHT LINE

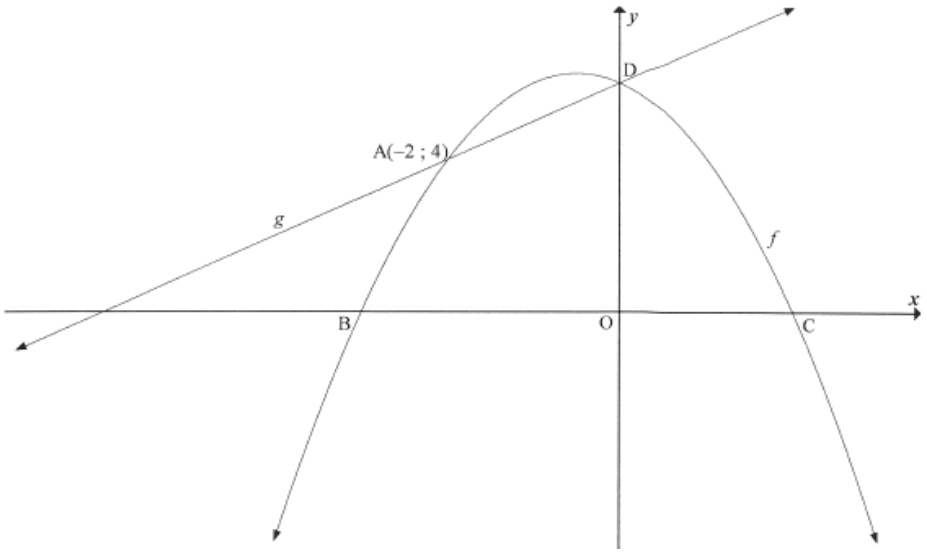
SUMMARY

WHAT YOU SHOULD KNOW

- All the terminology on page 4 and 5.
- Factorize a quadratic.
- Solve a quadratic equation.
- Formula for average gradient between points.
- How to determine the equation of a straight line.

QUESTION 2

The diagram shows the graphs of $f(x) = -x^2 - x + 6$ and $g(x) = mx + c$. $A(-2\,;4)$ is the point of intersection of the graphs.



2.1 Determine the x- intercepts of f .

2.2 Determine the equation of the axis of symmetry of f.

2.3 Determine the range of f.

2.4 Calculate down the average gradient between point A and D.

2.5 Write down the equation of g, in the form $g(x) = mx + c$.

2.6 Determine the equation of h if h is the reflection of f about the x- axis and then translated 3 units to the right. Leave your answer in the form $h(x) = a\,(x + p)^2 + q$.

2.7 Write down the values of x for which $x.f(x) < 0$.

2.8 If $f(p) = f(r) = 4$, calculate the value of $p - r$ if $r < 0$.

SESSION 1: PARABOLA AND STRAIGHT LINE

SUMMARY

WHAT YOU SHOULD KNOW

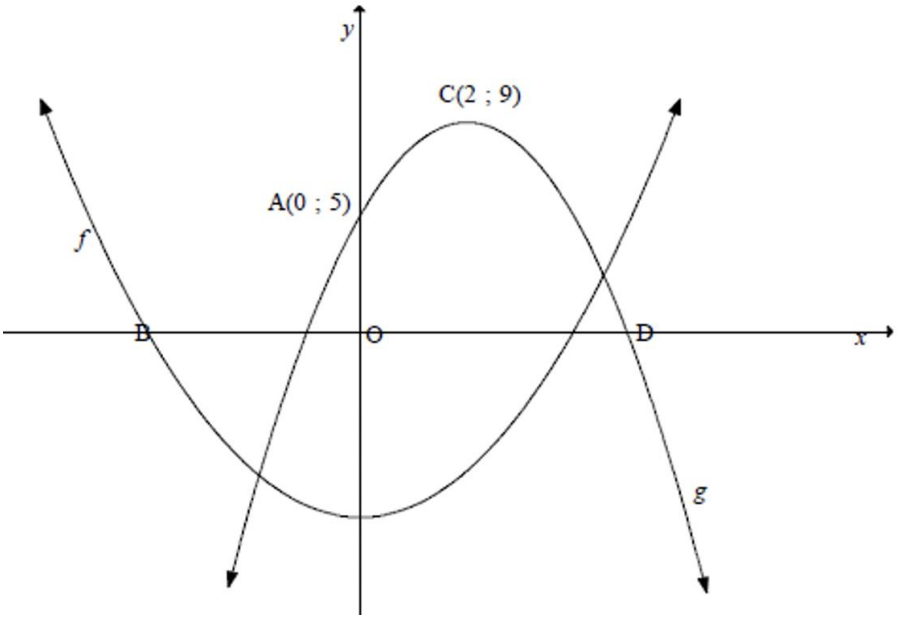
- All the terminology on page 4 and 5.
- Factorize a quadratic.
- Formula for average gradient.
- Turning point form of the equation of a parabola.

QUESTION 3

The sketch below represents the graphs of two parabolas, f and g.

$$f(x) = \frac{1}{2}x^2 - 8$$

The turning point of g is C(2 ; 9) and the y-intercept of g is A(0 ; 5). B and D are the x-intercepts of f and g respectively.



3.1 Show that $g(x) = -x^2 + 4x + 5$.

3.2 Calculate the average gradient of g between A and C.

3.3 Calculate the length of BD.

3.4 Use the graphs to solve for x, if:

3.4.1 $f(x) \geq 0$

3.4.2 f and g are both strictly increasing

QUESTION 4

Given : $f(x) - 2x^2 + x + 6$

4.1 Calculate the coordinates of the turning point of f.

4.2 Determine the y-intercept of f.

4.3 Determine the x-intercept of f.

4.4 Sketch the graph of f showing clearly all intercepts with the axes and turning point.

4.5 Determine the values of k such that $f(x) = k$ has equal roots.

4.6 If the graph of f is shifted two units to the right and one unit upwards to form h, determine the equation h in the form $y = a(x + p)^2 + q$.

SUMMARY

WHAT YOU SHOULD KNOW

- All the terminology on page 4 and 5.
- How to determine asymptotes of hyperbola.
- How to determine x- and y-intercepts of a function.
- Shape of hyperbola.
- Solve linear and exponential equations.
- Shape of the exponential function.
- Formula for average gradient.

QUESTION 5

Given: $f(x) = \frac{8}{x-2} + 3$

5.1 Write down the equations of the asymptotes of f.

5.2 Calculate the x- and y-intercepts of f.

5.3 Write down the domain.

5.4 Sketch the graph of f. Show clearly the intercepts with the axes and the asymptotes.

5.5 If $y = x + k$ is an equation of the line of symmetry of f, calculate the value of k.

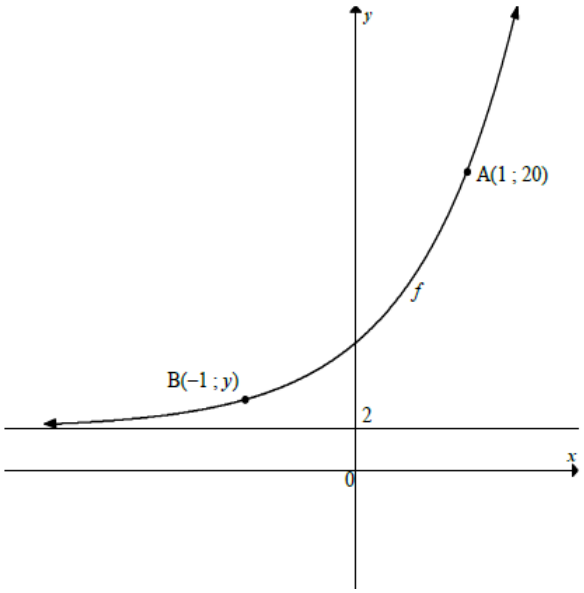
5.6 The graph of f is transformed to obtain the , $m(x) = \frac{-8}{x+1} + 3$. Describe the transformation.

QUESTION 6

The sketch given is the graph of $f(x) = 2.b^{x+1} + q$.

The graph of f passes through the points A(1 ; 20) and B (–1 ; y).

The line $y = 2$ is an asymptote of f.



6.1 Show that the equation of f is, $f(x) = 2(3)^{x+1} + 2$

6.2 Calculate the y-coordinate of the point B.

6.3 Determine the average gradient of the curve between the points A and B.

6.4 A new function h is obtained when f is reflected about its asymptote.

6.4.1 Determine the equation of h.

6.4.2 Write down the range of h.

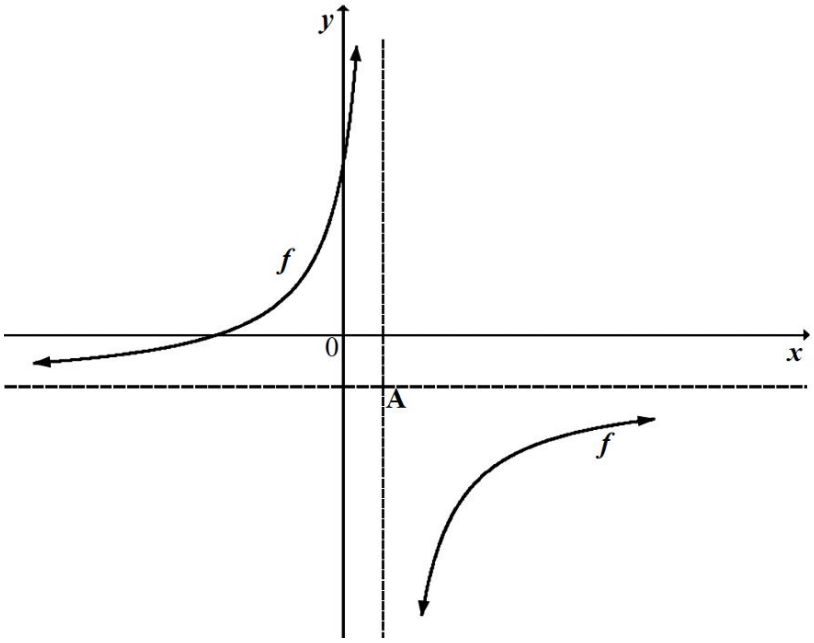
SUMMARY

WHAT YOU SHOULD KNOW

- All the terminology on page 4 and 5.
- How to solve a linear and exponential equation.
- Know how quadrants are labelled in the cartesian plane.
- Shape of exponential function.
- Hyperbola has two asymptotes.
- Shape of the exponential function.
- All the points on the x-axes has a y-coordinate of 0.

QUESTION 7

The sketch below shows the graph of
$f(x) = \frac{-9}{x-1} - 2$.
A is a point of intersection of the asymptotes of f.



7.1 Write down the coordinates of A.

7.2 Determine the coordinates of the x-intercepts of f.

7.3 Write down an equation of the axis of symmetry of f that has a negative gradient.

7.4 Hence or otherwise, determine the coordinates of a point that lies on f in the fourth quadrant, which is closest to point A.

7.5 The graph of f is reflected about the x- axis to obtain the graph of g.
Write down the equation of g in the form $y = \ldots\ldots\ldots\ldots\ldots\ldots$

8. Given : $h(x) = 4(2^{-x}) + 1$

8.1 Determine the coordinates of the y – intercept of h.

8.2 Explain why h does not have an x – intercept.

8.3 Draw a sketch graph of h, clearly showing all asymptotes, intercepts with the axes and at least one other point on h.

8.4 Describe the transformation from h to g if $g(x) = 4(2^{-x} + 2)$.

SUMMARY

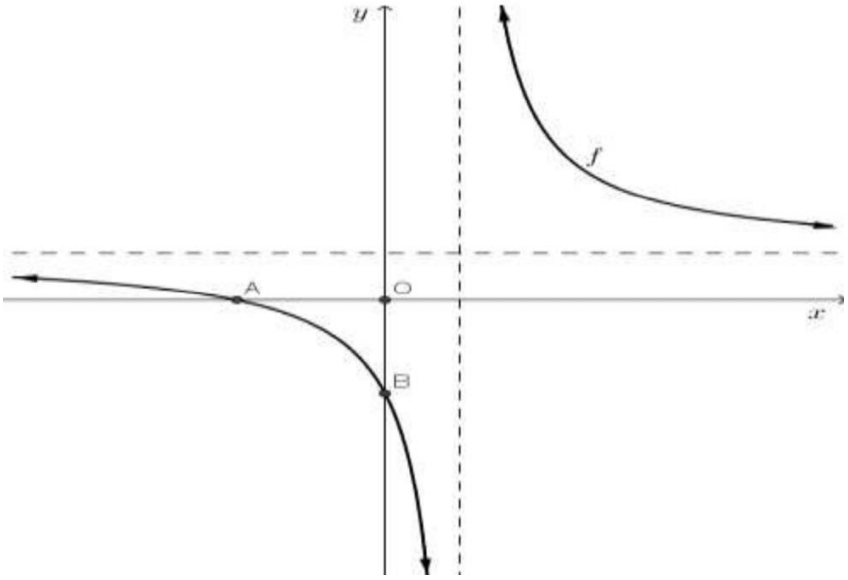
WHAT YOU SHOULD KNOW

- All the terminology on page 4 and 5.
- Formula for area of a triangle.
- All the points on the x-axes has a y-coordinate of 0.

QUESTION 9

Given is the sketch of:

$$f(x) = \frac{x+2}{x-1}$$



9.1 Write down the equation of the vertical asymptote of f.

9.2 Determine the coordinates of A.

9.3 Calculate the area of ΔAOB.

9.4 Show that $f(x)$ can be written as $f(x) = \frac{3}{x-1} + 1$

9.5 The graph of f is shifted such that point A lies on the origin. What are the coordinates of the point of intersection of the asymptotes of the new graph?