Telematic Schools Project

2022 SUBJECT WORKBOOK
Grade 10

$f(x)=x^2$ MATHEMATICS

A joint initiative between the Western Cape Education Department and Stellenbosch University.

Western Cape Government
Education

Stellenbosch
UNIVERSITY
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saam vorentoe

BROADCAST SESSIONS

GRADE 10 Algebra and Functions

Session	Date	Time	Topic
1	**14/04/2022**	**15h00-16h00**	**Algebra**
2	19/04/2022	15h00-16h00	Algebra
3	**03/08/2022**	**15h00-16h00**	**Functions: Parabole & Line**
4	04/08/2022	15h00-16h00	Funksies: Parabool en Reguit Lyn
5	**11/10/2022**	**15h00-16h00**	**Functions: Hyperbola**
6	12/10/2022	15h00-16h00	Funksies: Hiperbool
7	**31/10/2022**	**15h00-16h00**	**Trigonometry Functions**
8	01/11/2022	15h00-16h00	Trigonometriese Funksies

INTRODUCTION

Dear Grade 10 Learner

In 2022 there will be 8 live grade 10 Telematics sessions, 4 of these sessions is in English and 4 in Afrikaans.

This workbook contain material for all the Grade 10 sessions scheduled.

Please email, school@sun.ac.za with your name, surname, email and school name to register to obtain a username and password. You will then be able to access all previously recorded Telematics sessions and materials on other Topics from the Telematics Schools Project website, https://schools.sun.ac.za/ . Please note that accessing these recordings from this website is zero-rated.

In the Telematic lesson, the presenters will provide you with a summary of the important concepts and together with you they will work though some of the exercises. You are encouraged to come prepared, have a pen and enough paper (ideally a hard cover exercise book) and a scientific calculator.

You are also encouraged to participate fully in the lesson by asking questions and working out the exercises, and where you are asked to do so, sms or e-mail your answers to the studio.

Remember:" Success is not an event, it is the result of regular and consistent hard work".

GOODLUCK, Wishing you all the success you deserve!

SESSION 1: SOLVING QUADRATIC EQUATIONS

STANDARD FORM of quadratic equation: **:** $ax^2 + bx + c = 0$

TAKE NOTE : $a.b = 0$
$\Rightarrow a=0$ **or** $b=0$

- **Factorise** denominators, if any
- **Remove fractions** by multiplying each term with the LCM of the denominators
- Remove brackets (distributive law)
- Write equation in **standard form**
- Factorise quadratic
- Apply Rule: if *a.b = 0*
  $\Rightarrow$ *a=0* or *b=0*
- The solutions of a quadratic equation are called the **ROOTS** of the quadratic (max of 2 roots)

Solve for x:

Example 1: $(x-2)(x+3) = 0$

$\therefore x - 2 = 0$ or $x + 3 = 0$

$\therefore x = 2$ or $x = -3$

Example 2: $x^2 - 5x - 6 = 0$

$\therefore (x-6)(x+1) = 0$

$\therefore x - 6 = 0$ or $x + 1 = 0$

$\therefore x = 6$ or $x = -1$

Example 3: $x^2 = -5x$

$\therefore x^2 + 5x = 0$

$\therefore x(x+5) = 0$

$\therefore x = 0$ or $x + 5 = 0$

$\therefore x = 0$ or $\therefore x = -5$

NOT: $\div\, x$: $\therefore x = -5$
Never divide by the unknown/ variable; **dividing by zero**, is not allowed, this results in elliminating one of the solutions: **Write in standard form!!**

Example 4: $(2x-1)(x+2) = 25$

$\therefore 2x^2 + 3x - 2 - 25 = 0$

$\therefore 2x^2 + 3x - 27 = 0$

$\therefore (2x+9)(x-3) = 0$

$\therefore 2x + 9 = 0$ or $x - 3 = 0$

$\therefore x = -\frac{9}{2}$ or $x = 3$

NOT: $2x - 1 = 25$ **OR** $x + 2 = 25$
Rule only applicable if $a.b = 0$
Write in standard form!!

x^2 is **ALWAYS** ≥ 0

Example 5: $x^2 + 4 = 0$

$\therefore x^2 = -4$

$\therefore x = \pm\sqrt{-4}$

Non Real number

Answer: no $\mathbb{R}$ solution for x

Example 6: $\frac{x^2-2x-3}{x-3} = 2$

LCM $= (x-3)$; $x \neq 3$

$\times\,(x-3)$: $x^2 - 2x - 3 = 2(x-3)$

$\therefore x^2 - 2x - 3 = 2x - 6$

$\therefore x^2 - 4x + 3 = 0$

$\therefore (x+1)(x-3) = 0$

$\therefore x = -1$ or $\boxed{x = 3}$ Not applicable (not valid)

$\therefore x = -1$

WORKSHEET

Question 01

$$(x+3)(x-1)=0$$

Answer 01

Question 02

$$-2(x+2)(x+3)=0$$

Answer 02

Question 03

$$x^2+14x+48=0$$

Answer 03

Question 04

$$x^2-18=0$$

Answer 04

Question 05

$$(x-3)(x-2)=12$$

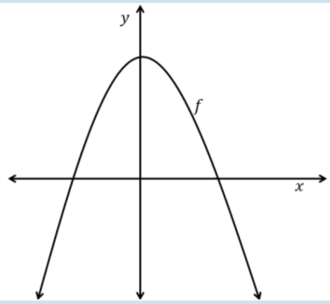
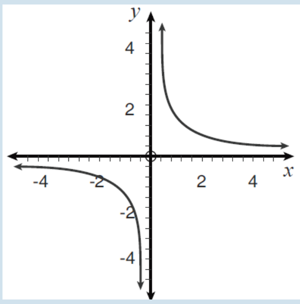
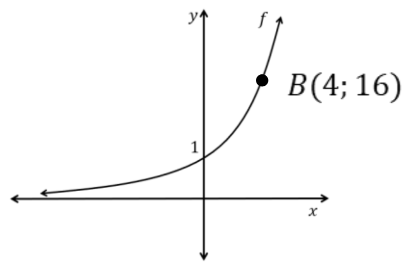
Answer 05

Question 06

$$\frac{x}{x-2}+\frac{2}{2-x}=\frac{1}{x-3}$$

Answer 06

SESSION 2 - 4:
TERMINOLOGY FUNCTIONS

Term	Definition
function	**A relation for which each element of the domain (x) corresponds to exactly one element of the range (y).** **For every x-value there is only one possible y-value.** **To test for a function on a graph, use the 'horizontal line test' – run a ruler from left to right. If your ruler only ever touches the graph in one spot, it is a function.**
parabola	
hyperbola	
exponential	
increasing function	A function that is going 'uphill' when looking at it from left to right. If a function has a turning point, this is the point at which a function could change from increasing to decreasing.
decreasing function	A function that is going 'downhill' when looking at it from left to right. If a function has a turning point, this is the point at which a function could change from decreasing to increasing.

SESSION 2 - 4: TERMINOLOGY FUNCTIONS

Term	Definition
horizontal shift	A translation of the graph either to the left or the right
vertical shift	A translation of the graph either up or down
average gradient	The gradient between two points on a curved graph.
domain	All the possible x-values that are covered by the function/graph. (To find the domain always study the graph from left to right)
range	All the possible y-values that are covered by the function/graph. (To find the range always study the graph from bottom to top)
x-intercept	Where a function crosses (intercepts) the x-axis
y-intercept	Where a function crosses (intercepts) the y-axis
axis of symmetry	A line that cuts the function exactly in half. A parabola has a vertical line of symmetry ($x = \ldots$) and a hyperbola has 2 axes of symmetry (which form a cross $\rightarrow X$. These are in the form $y = mx + c$) An exponential graph does not have an axis of symmetry
asymptote	A straight line that a curved graph gets closer and closer to but never touches it.
turning point	The point at which a function (parabola, sin or cos graph) changes from increasing to decreasing OR from decreasing to increasing
maximum	Only involves functions that have a turning point. The maximum is the highest the graph can be and is always the y-value of the turning point
minimum	Only involves functions that have a turning point. The minimum is the lowest the graph can be and is always the y-value of the turning point.
amplitude	The height from the centre line to the peak or trough
period	The distance required for the function to complete a full cycle. This is given in degrees.
parent function	the simplest function of a family of functions

SESSION 2:
The QUADRATIC FUNCTION (PARABOLA) $y = ax^2 + q$

- **DEFINITION OF A PARABOLA : A graph of a quadratic function i.e. $y = ax^2 + q$ where q is a real number and $a \neq 0$ is called a parabola**

The effect of the parameters $\boldsymbol{a}$ and $\boldsymbol{q}$ on the graph of the equation $\boldsymbol{y = ax^2 + q}$

If $\boldsymbol{a > 0}$, i.e.

if a is positive

The graph has

a **'smiley face.**

If $\boldsymbol{a < 0}$, i.e.

if a is negative

The graph has

a **frown.**

If $\boldsymbol{q > 0}$, i.e. if q is positive

Then q is the number of units the function,

$f(x) = x^2$ is shifted **up**

If $\boldsymbol{q < 0}$, i.e. if q is negative

Then q is the number of units the function,

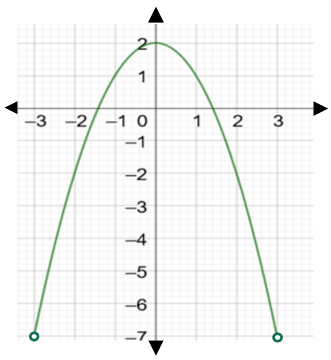
$f(x) = x^2$ is shifted **down**

Domain and Range of a Function

Example 1: Given

Determine a) domain

b) range



Domain: $-3 < x < 3$ or an interval $x \in (-3; 3)$

b) Range: $-7 < y \leq 2$ or $y \in (-7; 2]$

Domain are all x-values between -3 and 3, not including -3 and 3.

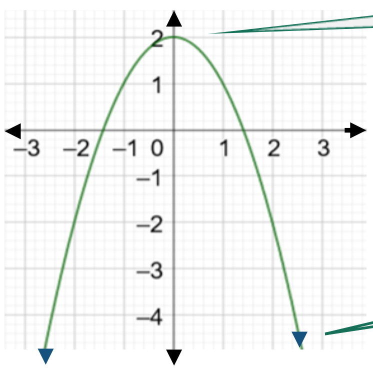
-7 is not included in the range of this function, because -3 and 3 is not in the domain of the function

As this parabola has open circles at both ends, the graph stops at these points with -3 and 3 not included as points on the graph.

Example 2: Given

Determine a) Range

b) Domain



a) Range: $y \leq 2$ or $y \in (-\infty; 2]$

Domain: $-\infty < x < \infty$ or $x \in R$

Note that the maximum value of this function is 2. Thus all real x-values will have a corresponding y-value smaller than 2. As this graph continues indefinitely the range is from 2 to $-\infty$.

This arrow indicate that the graph is continuing beyond this page. This function is defined for all real x-values.

$f(x)=x^2$

SESSION 2: SKETCH THE PARABOLA

- To sketch the parabola:

1. Establish the **shape of the graph** by identifying if the equation is of the form, $y = ax^2 + q$, if it is, you will know that it is a parabola. The sign of "a", will indicate that the parabola is either concave up or down.
2. Determine the **x- intercepts and y-intercepts**. Some parabolas do not have an x-intercept. If a parabola does not have an x- intercept, determine the coordinates of any other point on the graph and plot this on the sketch.
3. Determine the coordinates of the **Turning point.** The turning point for, $y = ax^2 + q$ is $(0; q)$. This point is also the y- intercept of this parabola.

Example 3

1. Sketch the graph of : $y = x^2 - 9$.
2. Write down the :
   2.1 domain
   2.2 range

3.1 Does the function have a maximum or minimum value?

3.2 Write down the equation of the axes of symmetry for he graph

Solution:

1. **Shape of graph** is: because $a > 0$ & form of equation Is $y = ax^2 + q$
2. **x-intercept**: let $y = 0$

$0 = x^2 - 9$

$0 = (x-3)(x+3)$

$(x-3) = 0 \text{ or } (x+3) = 0$

$x = 3 \text{ or } x = -3$

$(-3; 0) \text{ and } (3; 0)$

y-intercept: let $x = 0$

$y = (0)^2 - 9$

$y = -9 \therefore (0; -9)$

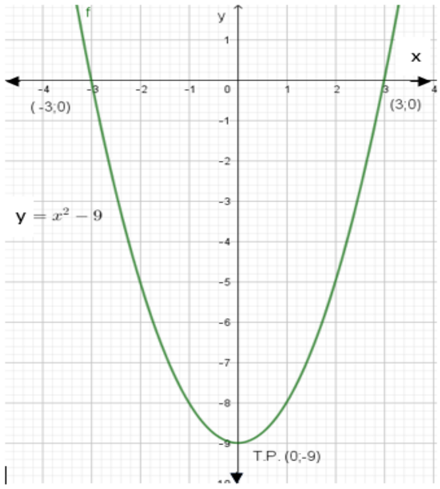
Turning Point: (0; -9)

2.1 Domain: $x \in R$

2.2 $-9 \leq y < \infty$ or $y \in [-9; \infty)$

3.1 Minimum Value of -9

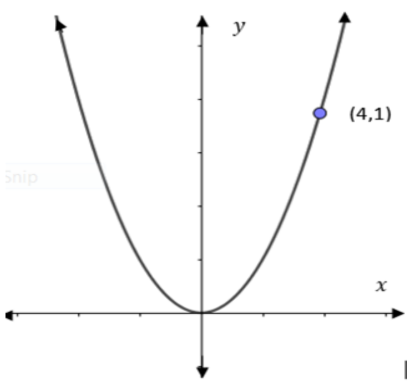
3.2 $x = 0$



When given a parabola of form $\boldsymbol{y = ax^2 + q}$. The **y-intercept** & **Turning point** is the same point. This point is: (0; q)

Determining the equation of a parabola : To find the equation of a parabola you need to find the values of a and q for the equation $y = ax^2 + q$

Example 4: Find the equation of the parabola below



Method of Solving

1. Identify the y-intercept/turning point, the y-coordinate will give you the value of q.
2. To find $\boldsymbol{a}$ we substitute the coordinates of a point on the graph into the equation.

Solution:

y-intercept (0; 0) $\therefore \quad q = 0$

$\therefore \quad y = ax^2 + 0$

Substitute the point $(4; 1)$ into $y = ax^2 \quad \therefore \quad 1 = a(4)^2$

$\therefore \quad 1 = 16a$

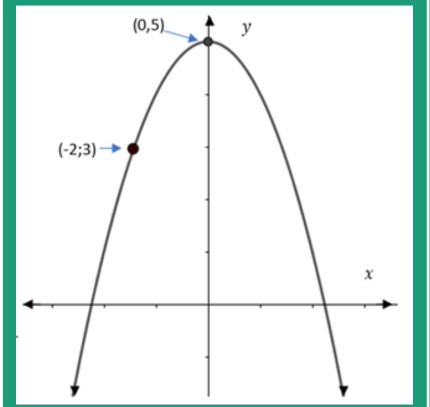
$(\div 16) \quad \therefore \quad a = \frac{1}{16}$

Substitute $a = \frac{1}{16}$ and $q = 0$ into the standard form of the equation

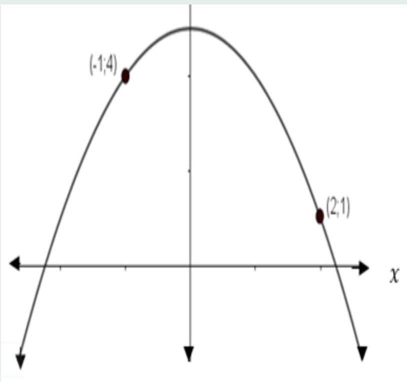
$\therefore$ The equation is $y = \frac{1}{16}x^2$

SESSION 2: DETERMINE THE EQUATION OF THE PARABOLA

Example 5: Determine the equation of the parabola passing through (0; 5) and (-2; 3).

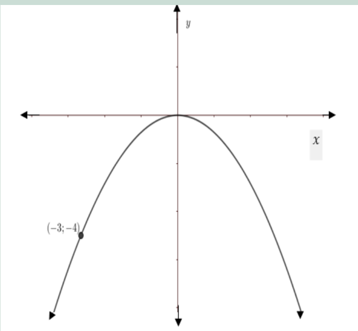


Example 6: Find the equation of the parabola which passes through the points (-1; 4) and (2; 1).

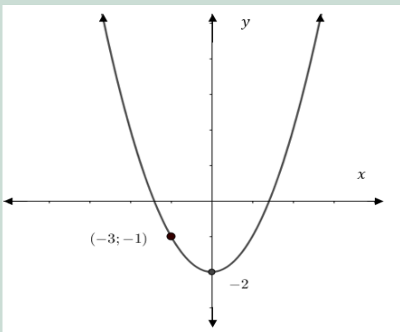


<u>QUESTION 1</u> Determine the equations of the following parabolas below

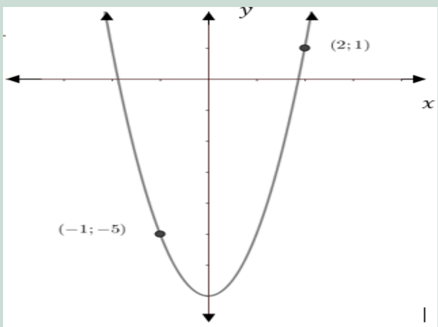
(a) The graph passing through (-3; -4)



(b) Through the point -2 and (-3; -1)



(c) Passing through the points (2;1) and (-1; -5)



SESSION 2: EXAMINATION TYPE QUESTIONS

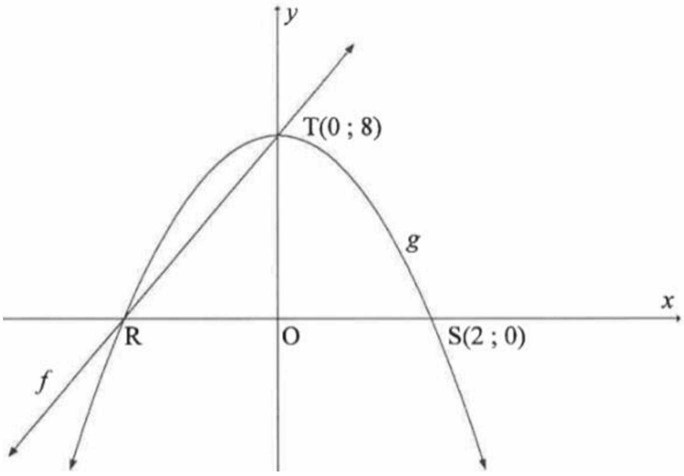
QUESTION 1

The diagram shows the graphs of

$g(x) = ax^2 + q$ and $f(x) = mx + c$.

R and S (2 ; 0) are the x-intercepts of g and T(0 ; 8) is the y- intercept of g.

Graph f passes through R and T.



1.1 Write down the range of g.

1.2 Write down the x- coordinate of R

1.3 Calculate the values of a and g.

1.4 Determine the equation of f .

1.5 Use the graphs to determine the value(s) of x for which:

1.5.1 $f(x) = g(x)$

1.5.2 $x.g(x) \leq 0$

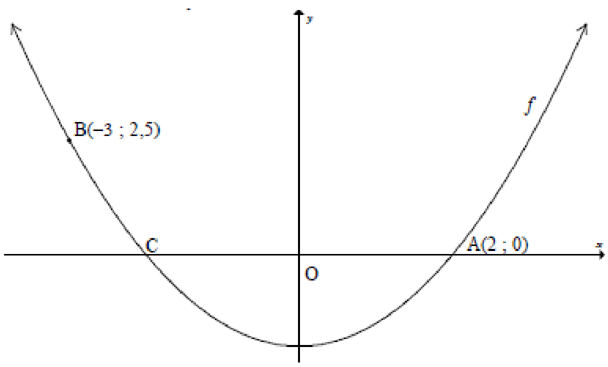
1.6 The graph h is obtained when g is reflected along the line $y = 0$.

Write down the equation of h in the form $h(x) = px^2 + k$.

QUESTION 2

The graph of $f\ (x) = ax^2 + q$ is sketched below.

Points A(2 ; 0) and B(– 3 ; 2,5) lie on the graph of f. Points A and C are x-intercepts of f.



2.1 Write down the coordinates of C.

2.2 Determine the equation of f.

2.3 Write down the range of f.

2.4 Write down the range of h, where $h(x) = -f(x) - 2$.

QUESTION 3

Given : $f(x) = ax^2 + c$

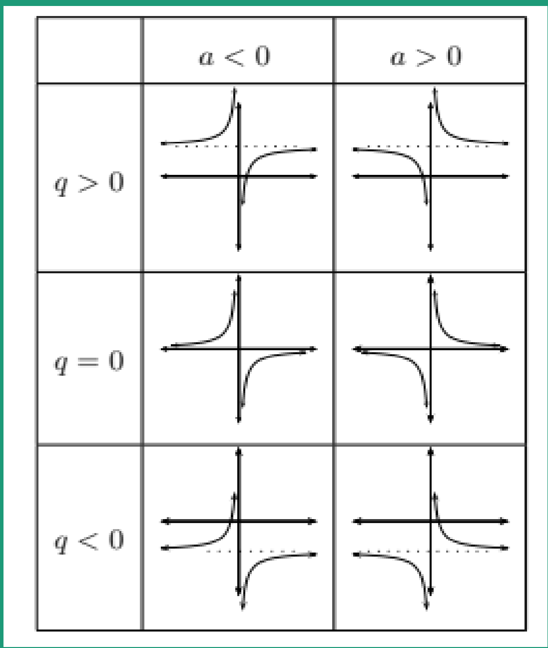
f passes through the x-axis at $(d-5)$ and $(d-1)$, where $d \in R$.

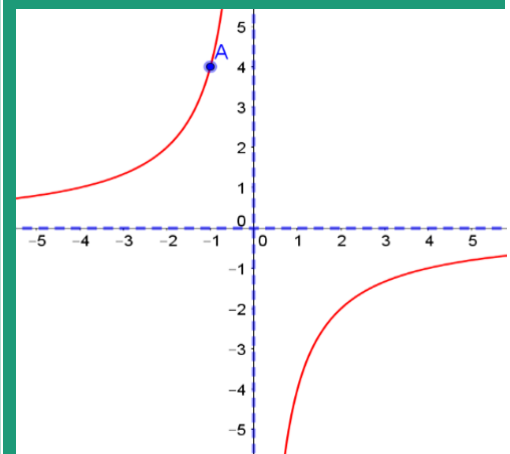
3.1 Determine the value of d.

3.2 Determine the values of a and c if it is also given that $f\ (1) = -9$.

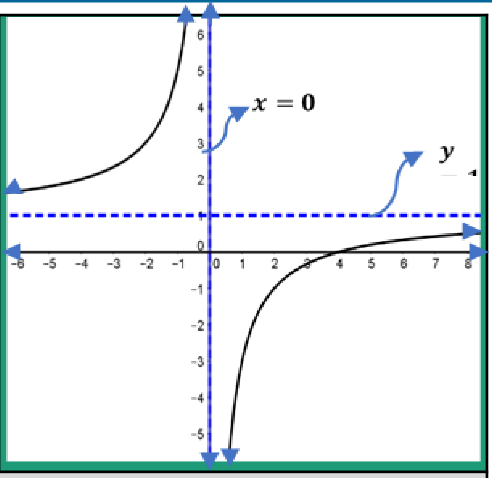
SESSION 3:

The Hyperbolic function $y = \frac{a}{x} + q$, Where $x \neq 0$ and $y \neq q$



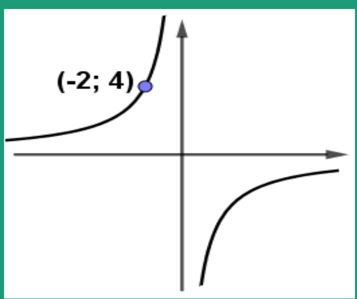


SESSION 3:
The Hyperbolic function $y = \frac{a}{x} + q$, Where $x \neq 0$ and $y \neq q$



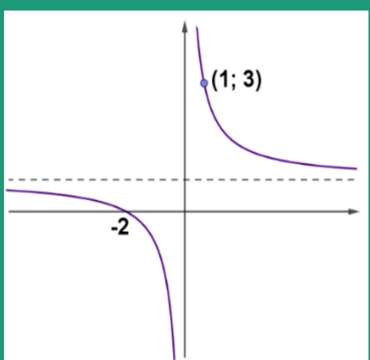
Finding the equation of the hyperbola.

Example 3.
Method 1:
Given the horizontal asymptote and or any other point on the graph.



Step 1: Use the general equation
Step 2: The horizontal asymptote gives you the value of q
Step 3: Substitute the coordinates of the point you have been given to find a

Example 4.
Method 2:
Given any two points on the graph



Step 1: Use the general equation
Step 2: Substitute the coordinates of the first point into the equation to give you equation 1.
Step 3: Substitute the coordinates of the second point into the equation to give you equation 2.
Step 4: Solve equation 1 and 2 simultaneously to find the values of a and q

SESSION 3: EXAMINATION TYPE QUESTIONS

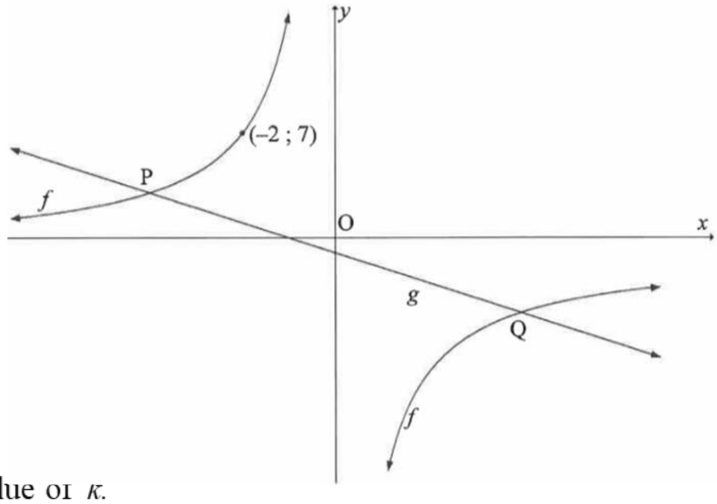
QUESTION 1

The sketch below shows the graphs of

$f(x) = \frac{k}{x} + w$ and $g(x) = -x - 1$.

The graph g is an axis of symmetry of f.

The graphs f and g intersect at P and Q.



1.1 Write down the value of w.

1.2 The point (-2; 7) lies on f. Calculate the value of k.

1.3 Calculate the x- coordinates of P and Q.

1.4 Write down the values of x for which $\frac{-16}{x} > x$

QUESTION 2

The sketch shows the graphs of

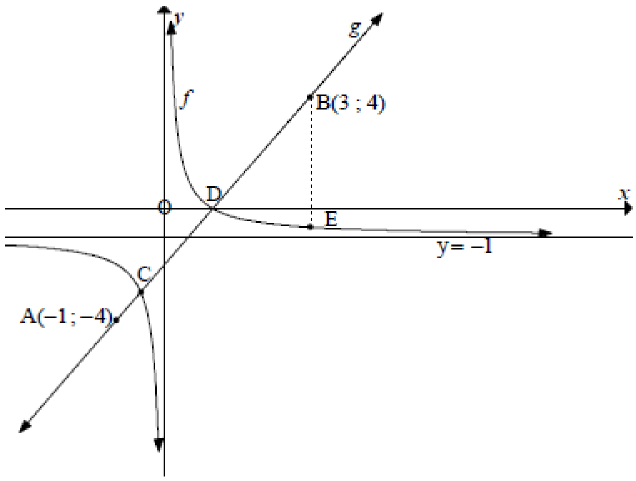
$f(x) = \frac{1}{x} - 1$ and $g(x) = ax + q$ respectively.

Point A(–1 ; –4) and B(3 ; 4) lie on the graph of g.

The two graphs intersect at points C and D.

Line BE is drawn parallel to the y-axis, with,

E on f.



2.1 Show that $a = 2$ and $q = -2$.

2.2 Determine the values x for which $f(x) = g(x)$.

2.3 For what values of x is $g(x) \geq f(x)$?

2.4 Calculate the length of BE.

2.5 Write down an equation of h if $h(x) = f(x) + 3$.

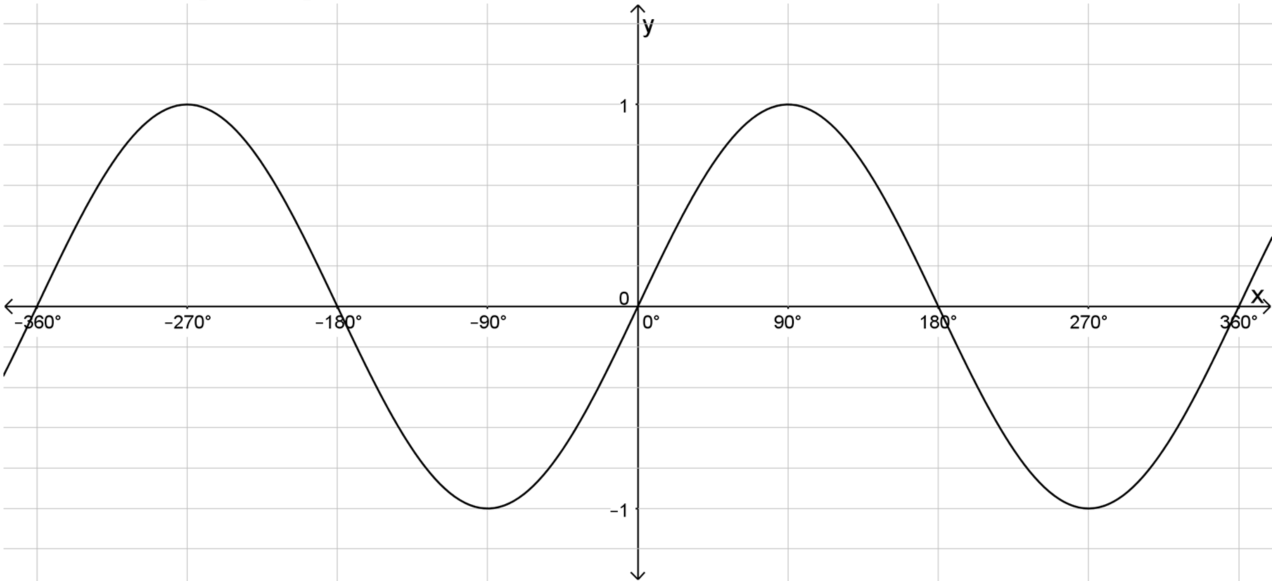
QUESTION 3

'n Hyperbola, h, is described with the following characteristics:

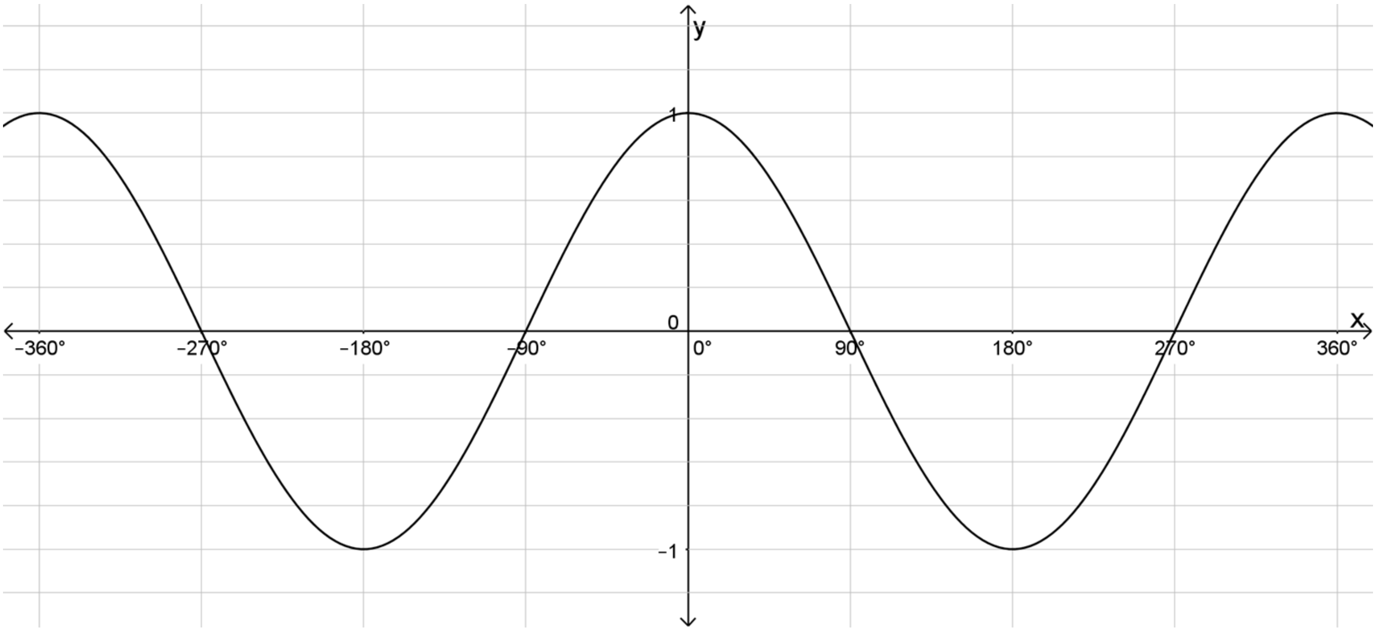
- The equation of the vertical asymptote is $x = 0$
- The range of h is $(-\infty ; 3) \cup (3 ; \infty)$
- The x-intercept of h is $(2 ; 0)$

Determine the equation of h.

Sine Curve:
$y = \sin x$ on interval $[-360°\,;360°]$ is sketched below.
Amplitude: 1
Period: $360°$
Domain: $x \in [-360°\,;360°]$ or $-360° \leq x \leq 360°$
Range: $y \in [-1\,;1]$ or $-1 \leq y \leq 1$



Cosine Curve:
$y = \cos x$ on interval $[-360°\,;360°]$ is sketched below.
Amplitude: 1
Period: $360°$
Domain: $x \in [-360°\,;360°]$ or $-360° \leq x \leq 360°$
Range: $y \in [-1\,;1]$ or $-1 \leq y \leq 1$



SESSION 4: TRIGONOMETRIC FUNCTIONS

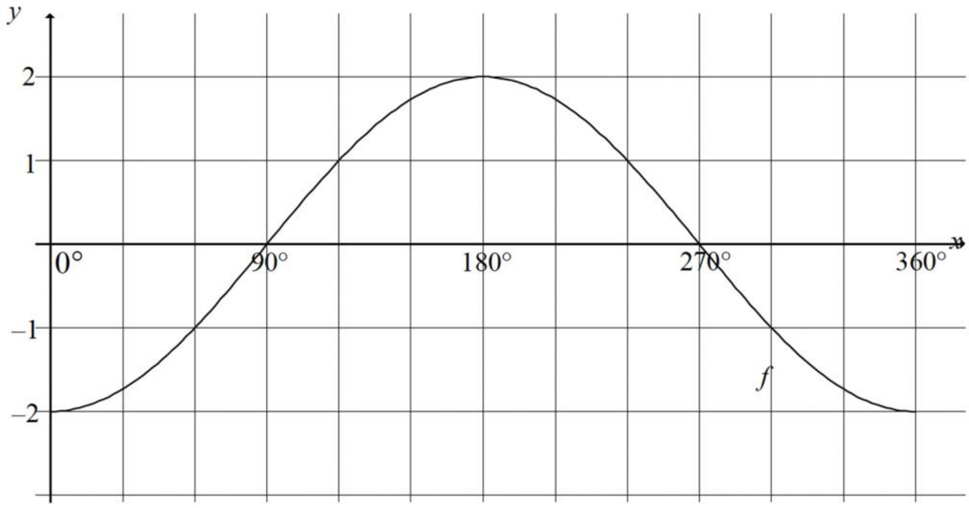
SUMMARY

WHAT YOU SHOULD KNOW

- Understand the information on page 15.
- Know how to sketch a Sine function.
- Known functional notation.
- Familiar with Transformations.
- How to use graphs to solve equations and inequalities.

QUESTION 1

The diagram, the graphs of $f(x) = -2\cos x$ is drawn for the interval $0° \leq x \leq 360°$.



1.1 Write down the amplitude of f .

1.2 Write down the minimum value of $f(x) + 3$.

1.3 On the same system of axes, draw the graph of g, where $g(x) = \sin x + 1$ for the interval $0° \leq x \leq 360°$.

1.4 Use the graphs to determine the following:

1.4.1 The value of $f(180°) - g(180°)$

1.4.2 For which value(s) of x will $f(x).g(x) > 0$

1.5 The graph of f is reflected about the x – axis and then moved 3 units downwards to form the graph of h. Determine:

1.5.1 The equation of h

1.5.2 The range of h for the interval $0° \leq x \leq 360°$.

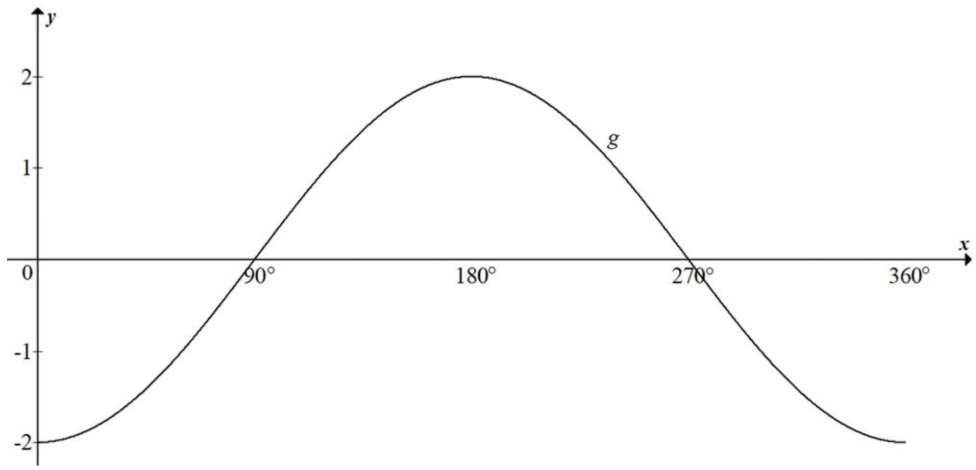
SUMMARY

WHAT YOU SHOULD KNOW

- Understand the information on page 15.
- Know how to sketch a Sine and cosine function.
- Known functional notation.
- Familiar with Transformations.
- How to use graphs to solve equations and inequalities.
- .

QUESTION 2

Sketched is the graph of $g(x) = a.\cos b\theta$



2.1 Write down the values of a and b.

2.2 Use the of the graph to determine the value(s) of x for which $g(x) > 0$.

2.3 Determine the range of h, if h is the image of g, if g is shifted down two units.

2.4 Determine, using the graph, the value of:

$$-2(\cos 0° + \cos 1° + \cos 2° + \ ... + \cos 358° + \cos 359° + \cos 360°)$$

QUESTION 3

Given $f(x) = \sin x - 1$ and $g(x) = 2\cos x$ for $0° \leq x \leq 270°$.

3.1 Sketch, on the grid provided in the Answer Book, the graph of f and g for $0° \leq x \leq 270°$.

3.2 Write down the following:

3.2.1 Amplitude of g

3.2.2 Range of f

3.3 Use your graph to determine the following:

3.3.1 Number of solutions to $f(x) = g(x)$ in the interval $0° \leq x \leq 270°$.

3.3.2 Value(s) of x in the interval $0° \leq x \leq 180°$ for which $\sin x = 2 + 2\cos x$.

SESSION 4 : TRIGONOMETRIC FUNCTIONS

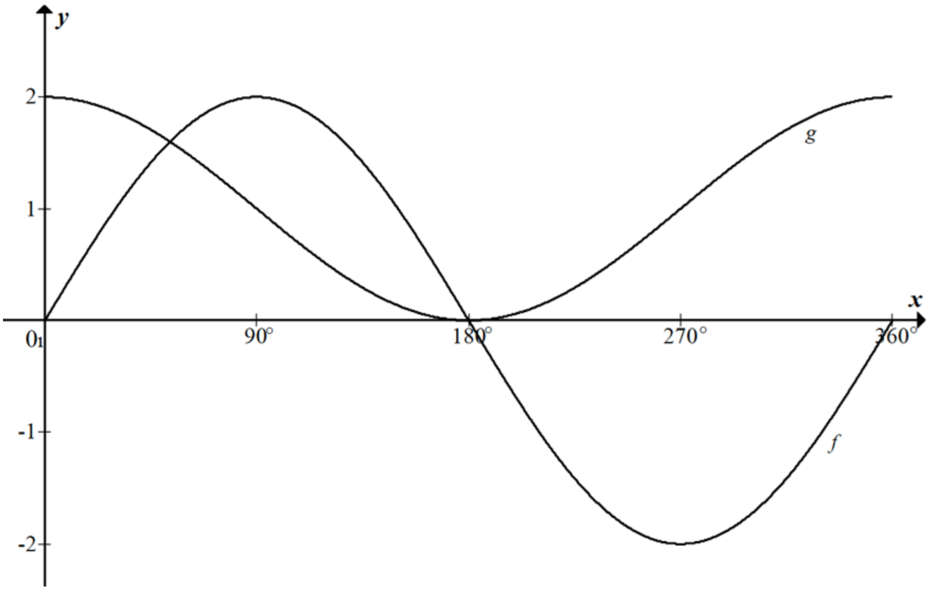
SUMMARY

WHAT YOU SHOULD KNOW

- Understand the information on page 15.
- Known functional notation.
- Familiar with Transformations.
- How to use graphs to solve equations and inequalities.

QUESTION 4

The graphs of $f(x) = a\sin x$ and $g(x) = \cos x + 1$ for $x \in [0° ; 360°]$ are sketched below.



4.1 Write down the value of a.

4.2 Write down the period of f.

4.3 Write down the range of g.

4.4 For which values of x for $x \in [0° ; 360°]$ will $f(x).g(x) > 0$.

4.5 The graph of g, is reflected about the x –axis and then shifted 2 units upwards to obtain the graph of h. Write down the equation of h.