Western Cape Government
Education

Directorate: Curriculum FET

TELEMATIC SCHOOLS PROJECT 2021

MATHEMATICS
Grade 12

Dear Grade 12 Learner

In 2021 there will be 10 live Telematics sessions, 5 of these sessions are in English and 5 in Afrikaans. Only one of the five sessions is indicated for grade 12. I would however like to recommend that you attend all the Telematics sessions as you will benefit from all of them. This is why on the next page the dates, times and topics for all sessions are included.

This workbook contain material for:

1. Grade 12 session on number patterns (17 March)
2. Grade 11 session on the quadratic number pattern (20 April)
3. Grade 11 session on Financial Mathematics (27 July)

Please note that you will be assessed on all the grade 11 content in grade 12.

Please email, school@sun.ac.za with your name, surname, email and school name to register to obtain a username and password. You will then be able to access all previously recorded Telematics sessions and materials on other Topics from the Telematics Schools Project website, https://schools.sun.ac.za/ . Please note that accessing these recordings from this website is zero-rated.

At school you should receive a book called "Grade 12 Tips for Success". In it you will have a breakdown of the weighting of the various Topics in Mathematics. Ensure that you download a QR reader, this will enable you to scan the various QR codes.

The "MATHEMATICS EXAMINATION GUIDELINE", can be downloaded by scanning the adjacent QR code or by clicking on, http://bit.ly/EXAM_GUIDE . This examination guideline document contains important information regarding your end of year examination paper as well as the acceptable abbreviations to Geometry theorems which you are to use when answering Geometry questions.

At the start of each Telematics lesson, the presenters will provide you with a summary of the important concepts and together with you will work though the exercises. You are encouraged to come prepared, have a pen and enough paper (ideally a hard cover exercise book) and your scientific calculator with you.

You are also encouraged to participate fully in each lesson by asking questions and working out the exercises, where you are asked to. Please, sms or e-mail your answers to the studio where you are asked to do so.

Remember:" Success is not an event, it is the result of regular and consistent hard work".

GOODLUCK, Wishing you all the success you deserve!

2021 Mathematics Telematics Program

Day	Date	Time	Grade	Subject	Topic
Term 1: 13 Jan - 26 March					
Wednesday	03 March	15:00 – 16:00	10	Mathematics	Algebra
Thursday	04 March	15:00 – 16:00	10	Wiskunde	Algebra
Wednesday	17 March	15:00 – 16:00	12	Mathematics	Number Patterns
Thursday	18 March	15:00 – 16:00	12	Wiskunde	Getalpatrone
Term 2: 13 April – 25 June					
Tuesday	20 April	15:00 – 16:00	11	Mathematics	Quadratic Number Patterns
Wednesday	21 April	15:00 – 16:00	11	Wiskunde	Kwadratiese Getalpatrone
Term 3: 20 July – 01 October					
Tuesday	27 July	15:00 – 16:00	11	Mathematics	Finance
Wednesday	28 July	15:00 – 16:00	11	Wiskunde	Finansies
Term 4: 12 October - 8 December					
Wednesday	20 October	16:00 – 17:00	11	*Mathematics*	*Exam Preparation*
Thursday	21 October	16:00 – 17:00	*11*	*Wiskunde*	*Eksamen Voorbereiding*

NUMBER PATTERNS, SEQUENCES AND SERIES

NUMBER PATTERNS:

Number Patterns are sequences of numbers that follow a particular rule. This rule, T_n is the *n'th* term of the sequence/number pattern in terms of the term number (*n)* .

In earlier grades you have encountered various number patterns. In grade 11 you have dealt with the quadratic number pattern. The quadratic number pattern is one having a constant second common difference.

In grade 12 the focus is on:

1. Arithmetic and Geometric number patterns or sequences
2. Arithmetic and Geometric series
3. Sigma Notation.

The quadratic number pattens will also be assessed in grade 12. The derivation of the formula to the sum of an Arithmetic Sequence as well as the sum to a Geometric Sequence must be known as it can be assessed.

<table>
<tr><th>Arithmetic Sequence (Linear Pattern):</th><th>Geometric Sequence (Exponential Pattern)</th></tr>
<tr><td>Arithmetic Sequence is a sequence of numbers where there is a constant common first difference(d). Consider the sequence,
$a\ ;\ a+d\ ;\ a+2d\ ;\ \ldots\ldots\ldots$
$\therefore \quad d = T_2 - T_1 = T_3 - T_2$
And $T_n = a + (n-1)d$</td><td>A Geometric Sequence is a sequence of numbers where there is a constant common ratio(r). Consider the sequence,
$a\ ;\ ar\ ;\ ar^2\ ;\ \ldots\ldots\ldots$
$\therefore \quad r = \frac{T_2}{T_1} = \frac{T_3}{T_2}$
And $T_n = ar^{n-1}$</td></tr>
<tr><th>Arithmetic Series</th><th>Geometric Series</th></tr>
<tr><td>Arithmetic Series is the sum of the terms of an Arithmetic Sequence.
$a + (a+d) + (a+2d) + \cdots\ldots\ldots$
The sum of an arithmetic sequence can be calculated using the formula:
$S_n = \frac{n}{2}(2a + (n-1)d)$</td><td>Geometric Series is the sum of the terms of a Geometric Sequence.
$a + (ar) + (ar^2) + \ \ldots\ldots\ldots$
The sum of a Geometric sequence can be calculated using the formula:
$S_n = \frac{a(r^n-1)}{r-1}$ Or $S_n = \frac{a(1-r^n)}{1-r}$ and $r \neq 1$</td></tr>
<tr><td rowspan="2"></td><th>Convergent Geometric Series</th></tr>
<tr><td>A geometric series will converge if $-1 < r < 1$.
The sum to infinity can be calculated of a Geometric Sequence/Series if $-1 < r < 1$.
$S_\infty = \frac{a}{1-r}$</td></tr>
<tr><th>Other Sequence or Series</th><th>Sigma Notation ($\sum$)</th></tr>
<tr><td>Other patterns can be created by combining,
1. An Arithmetic with a Geometric Pattern
2. Arithmetic or Geometric with a constant pattern
3. Arithmetic or Geometric Pattern with some other pattern
Example:
$\frac{2}{1\times 2}\ ;\ \frac{2}{2\times 3}\ ;\ \frac{2}{3\times 4}\ ;\ \ldots\ldots$</td><td>Sigma notation is a very useful notation for the sum of a given number of terms of a sequence.
$\sum$ is the symbol used for, 'sum of '
$\sum_{k=3}^{5} 3k$
1. this is read as the sum of all the terms from the 3rd term to the 5th term of the sequence.
2. It can be expanded as,
$\sum_{k=3}^{5} 3k = 3(3) + 3(4) + 3(5) = 9 + 12 + 15 = 36$</td></tr>
</table>

Exercises:

1. Given the sequence 6; 13; 20; …
   a) Determine a formula for the n^{th} term of the sequence.
   b) Calculate the 21^{st} term of this sequence.
   c) Determine which term of this sequence is 97.

2. Consider this number pattern: 8; 5; 2; …
   a) Calculate the 15^{th} term.
   b) Determine which term of this sequence is –289.

3. a) Given the arithmetic sequence $1 - p; 2p - 3; p + 5; \ldots$ determine the value of p.
   b) Determine the values of the first three terms of the sequence.

4. a) Determine the 10^{th} term of the sequence: 3; 6; 12; . . .
   b) Determine the number of terms in the sequence: 2; 4; 8; . . .; 1024
   c) If 5; x; 45 are the first three terms of a geometric sequence, determine the value of x.
   d) Determine the geometric sequence whose 8^{th} term is 9 and whose 10^{th} term is 25.

5. Determine the sum of the series: 19 + 22 + 25 + . . . + 121

6. The sum of the series 22 + 28 + 34 + . . . is 1870. Determine the number of terms.

7. Given the arithmetic sequence -3; 1; 5; …,393
   a) Determine a formula for the n^{th} term of the sequence.
   b) Write down the 4^{th}, 5^{th}, 6^{th} and 7^{th} terms of the sequence.
   c) Write down the remainders when each of the first seven terms of the sequence is divided by 3.
   d) Calculate the sum of the terms in the arithmetic sequence that are divisible by 3.

8. The sum of n terms is given by $S_n = \frac{n}{2}(1 + n)$. Determine T_5.

9. $3x + 1;\ 2x;\ 3x - 7$ are the first three terms of an arithmetic sequence. Calculate the value of x.

10. The first and second terms of an arithmetic sequence are 10 and 6 respectively.
    a) Calculate the 11^{th} term of the sequence.
    b) The sum of the first n terms of this sequence is –560. Calculate n.

11. Determine 3 + 6 + 12 + 24 + . . . to 10 terms.

12. If 2 + 6 + 18 + . . . = 728, determine the value of n.

13. Evaluate:

$$\sum_{n=4}^{70}(2n - 4)$$

14. What is the value of m for which:

$$\sum_{k=1}^{m} 5(3)^{k-1} = 65$$

15. Consider the sequence:

$$\frac{1}{2}\ ;4\ ;\frac{1}{4}\ ;7\ ;\frac{1}{8};10\ ;\ldots\ldots$$

a) If the pattern continues in the same way, write down the next Two terms in the sequence.
b) Calculate the sum of the first 50 terms of the sequence.

16. Calculate S_∞ if

$$\sum_{p=1}^{\infty} 8(4)^{1-p}$$

17. Given the series: $3(2x-3)^2 + 3(2x-2)^3 + 3\,(2x-3)^4 + \cdots$. for which vakues of x will the series converge?

18. Find the value of m if:

$$\sum_{k=1}^{m} 3(2)^{k-1} = 93$$

19. For which values of x will

$$\sum_{k=1}^{\infty} (4x-1)^k$$

exist.

Questions from National Examination Papers:

QUESTION 1 DBE/November 2019

1.1 Given the quadratic sequence: 321 ; 290 ; 261 ; 234 ; ….

1.1.1 Write down the values of the next TWO terms of the sequence. (2)

1.1.2 Determine the general term of the sequence in the form $T_n = an^2 + bn + c$. (4)

1.1.3 Which term(s) of the sequence will have a value of 74? (4)

1.1.4 Which term in the sequence has the least value? (2)

1.2 Given the geometric series: $\frac{5}{8} + \frac{5}{16} + \frac{5}{32} + \ldots = \text{K}$

1.2.1 Determine the value of K if the series has 21 terms. (3)

1.2.2 Determine the largest value of n for which $T_n > \frac{5}{8192}$ (4)

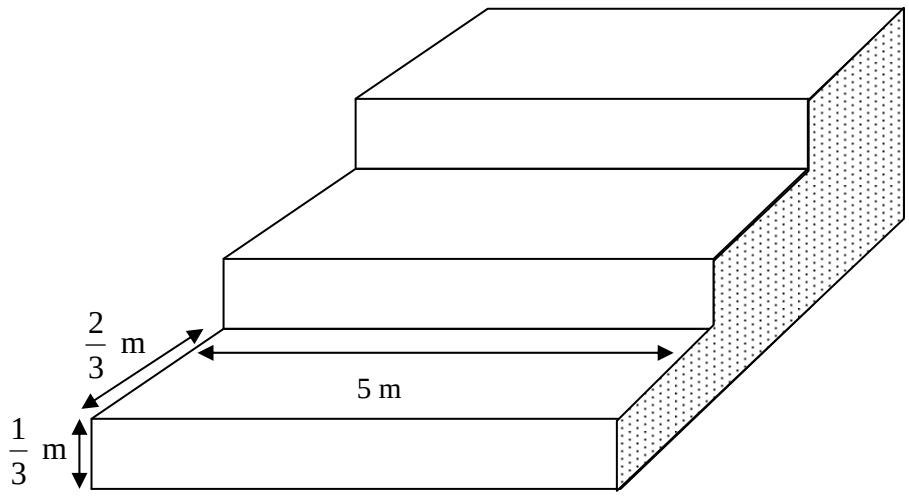
[19]

QUESTION 2 DBE/November 2019

2.1 Without using a calculator, determine the value of: $\sum_{y=3}^{10} \frac{1}{y-2} - \sum_{y=3}^{10} \frac{1}{y-1}$ (3)

2.2 A steel pavilion at a sports ground comprises of a series of 12 steps, of which the first 3 are shown in the diagram below.

Each step is 5 m wide. Each step has a rise of $\frac{1}{3}$ m and has a tread of $\frac{2}{3}$ m, as shown in the diagram below.



The open side (shaded on sketch) on each side of the pavilion must be covered with metal sheeting. Calculate the area (in m^2) of metal sheeting needed to cover both open sides. (6)

[9]

QUESTION 3 DBE/November 2018

3.1 Given the quadratic sequence: 2 ; 3 ; 10 ; 23 ; …

3.1.1 Write down the next term of the sequence. (1)

3.1.2 Determine the n^{th} term of the sequence. (4)

3.1.3 Calculate the 20^{th} term of the sequence. (2)

3.2 Given the arithmetic sequence: 35 ; 28 ; 21 ; …

Calculate which term of the sequence will have a value of –140. (3)

3.3 For which value of n will the sum of the first n terms of the arithmetic sequence in QUESTION 2.2 be equal to the n^{th} term of the quadratic sequence in QUESTION 2.1? (6)

[16]

DBE/November 2018

QUESTION 4

A geometric series has a constant ratio of $\frac{1}{2}$ and a sum to infinity of 6.

4.1 Calculate the first term of the series. (2)

4.2 Calculate the 8^{th} term of the series. (2)

4.3 Given: $\sum_{k=1}^{n} 3(2)^{1-k} = 5{,}8125$ Calculate the value of n. (4)

4.4 If $\sum_{k=1}^{20} 3(2)^{1-k} = p$, write down $\sum_{k=1}^{20} 24(2)^{-k}$ in terms of p. (3)

[11]

QUESTION 5

DBE/November 2017

5.1 Given the following quadratic number pattern: $5 \; ; \; -4 \; ; \; -19 \; ; \; -40$

5.1.1 Determine the constant second difference of the sequence. (2)

5.1.2 Determine the n^{th} term (T_n) of the pattern. (4)

5.1.3 Which term of the pattern will be equal to $-25\,939$? (3)

5.2 The first three terms of an arithmetic sequence are $2k-7 \; ; \; k+8$ and $2k-1$.

5.2.1 Calculate the value of the 15^{th} term of the sequence. (5)

5.2.2 Calculate the sum of the first 30 even terms of the sequence. (4)

[18]

QUESTION 6

A convergent geometric series consisting of only positive terms has first term a, constant ratio r and n^{th} term, T_n, such that $\sum_{n=3}^{\infty} T_n = \frac{1}{4}$.

6.1 If $T_1 + T_2 = 2$, write down an expression for a in terms of r. (2)

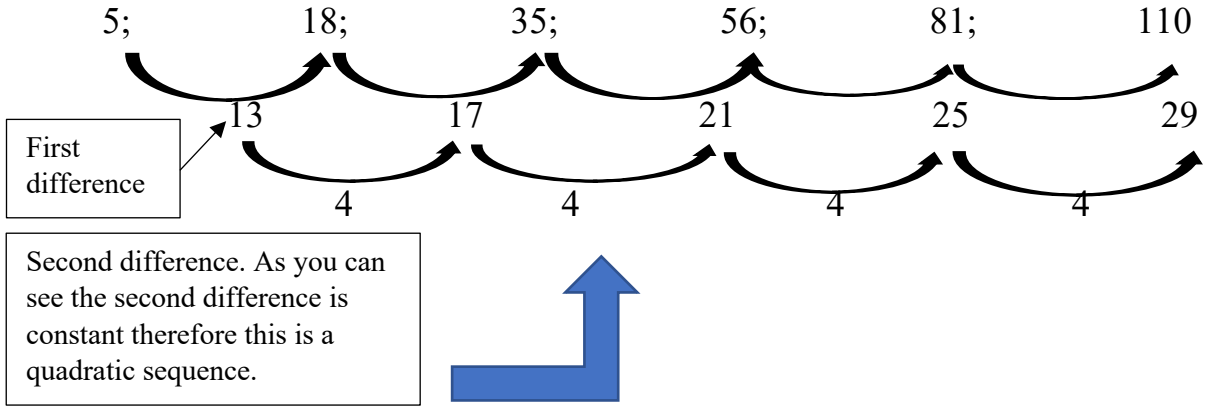
6.2 Calculate the values of a and r. (6)

[8]

Quadratic Number Patterns (Grade 11: 20 April 2021)

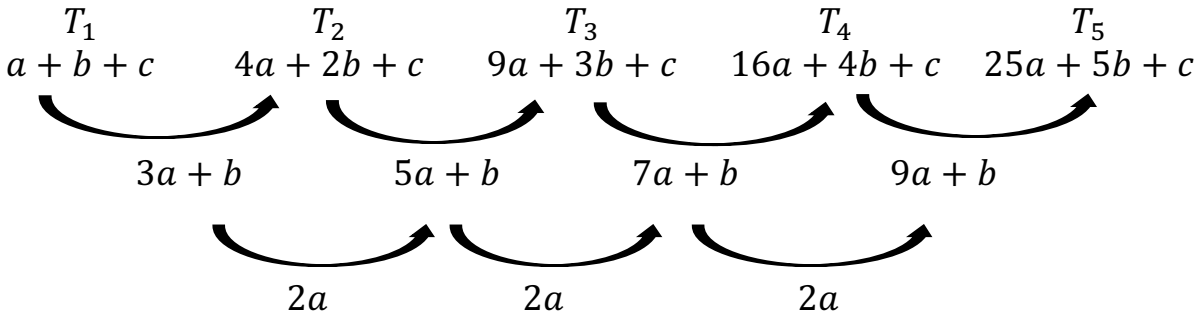
Definition:	**General term:**
A quadratic number pattern is formed when the second difference of a sequence is a constant value.	$T_n = an^2 + bn + c$

Example 1:



Example 2:

Substitute the first five terms into the general term.



The substitution of five terms into the quadratic general formula helps us to arrive at a method to determine a, b and c of the general formula. Let's see how that works:

- The second constant difference is equal to $2a$. Therefore, with algebraic manipulation, one can state that $a = \frac{2^{nd}constant\ difference}{2}$.
- $T_2 - T_1 = 3a + b$
  $\therefore b = T_2 - T_1 - 3a$

  Sub in T_1, T_2 and a
- $T_1 = a + b + c$
  $\therefore c = T_1 - a - b$

  Sub in T_1, a and b

Let's find the general term of the quadratic sequence in example 1:

Given: 5; 18; 35; 56; 81; 110

$a = \frac{2^{nd}constant\ difference}{2}$ $\quad \therefore a = \frac{4}{2} = 2$

$b = T_2 - T_1 - 3a$ $\quad \therefore b = 18 - 5 - 3(2) = 7$

$c = T_1 - a - b$ $\quad \therefore c = 5 - 2 - 7 = -4$

$\therefore T_n = 4n^2 + 7n - 4$

Questions from Examination Papers

Question 1 (Gr 11/ November 2018)

The quadratic number pattern: $4 ; p ; 11 ; q ; 22 ; ....$ has a constant second difference of 1.

1.1 Show that $p = 7$ and $q = 16$

1.2 Determine the general term, T_n, of the quadratic pattern.

1.3 Determine the value of n if $T_n = 232$

1.4 If the sum of two consecutive terms in the pattern is 1227, calculate the difference between these two terms.

Question 2 (Gr 11/ November 2017)

Given the quadratic pattern: 244 ; 193 ; 148 ; 109 ;…….

2.1 Write down the next term of the pattern.

2.2 Determine a formula for the n^{th} term of the pattern.

2.3 Which term of the pattern will have a value of 508?

2.4 Between which TWO consecutive terms of the quadratic pattern will the first difference be 453?

2.5 Show that all the terms of the quadratic pattern are positive.

Question 3 (Gr 11/ November 2016)

Consider the quadratic pattern: – 9; – 6 ; 1 ; 12 ; x ; …

3.1 Determine the value of x.

3.2 Determine a formula for the n^{th} term of the pattern.

3.3 A new pattern, P_n, is formed by adding 3 to each term in the given quadratic pattern. Write down the general term of P_n in the form $P_n = an^2 + bn + c$.

3.4 Which term of the sequence found in Question 3.3 has a value of 400?

Question 4 (Gr 11/ November 2016)

4.1 A quadratic pattern has a constant second difference of 2 and $T_5 = T_{17} = 29$

4.1.1 Does this pattern have a minimum or maximum value? Justify your answer.

4.1.2 Determine an expression for the n^{th} term in the form, $T_n = an^2 + bn + c$

Question 5 (Gr 11/ November 2015)

5.1 Consider the quadratic pattern: 5 ; 12 ; 29 ; 56 ; …

5.1.1 Write down the NEXT TWO terms of the pattern.

5.1.2 Prove that the first differences of this pattern will always be odd.

Question 6 (Gr 11/ November 2015)

6.1 Consider the quadratic pattern: 3 ; 5 ; 8 ; 12 ; …

Determine the value of T_{26}.

6.2 A certain quadratic pattern has the following characteristics:

- $T_1 = p$
- $T_2 = 18$
- $T_4 = 4T_1$
- $T_3 - T_2 = 10$

Determine the value of p.

Financial Mathematics (Gr 11: 27 July2021)

Financial terms:

- Interest is a fee paid for the use of borrowed money, or money earned on money saved. It is calculated as a percentage of the money borrowed or lent.
- Simple interest is the interest on an initial (principal) sum of money. Each year you receive or you are charged the same amount of interest.
- Compound interest is also interest on a principal amount P. For each year, the previous year's final amount becomes the new principal amount. So the interest is calculated on the principal and the interest from the previous year.
- Decay or depreciation is when a quantity decreases by a percentage of the amount present. For example, your assets (house, car) and machinery lose value through age and use.
- A nominal interest rate is the quoted interest rate.
- An effective interest rate is the actual interest rate received. If you are quoted a nominal interest rate of 7% p.a., the resulting effective rate will be different depending on if it is worked out monthly or quarterly.

Simple interest: $A = P(1 + ni)$	**Compound interest**: $A = P(1 + i)^n$
Simple decay or depreciation (Straight line depreciation): $A = P(1 - ni)$	**Compound decay or depreciation** (reducing balance depreciation): $A = P(1 - i)^n$

Where

P is the principal (original sum of money invested or borrowed)

i is the interest (rate per period)

n is the number of years (no of periods)

A is the final amount

Nominal and effective interest rates: This formula is used to either calculate the nominal or effective interest rate, where k is the number of times per year interest is calculated.

$$1 + i^{effective} = \left(1 + \frac{i^{nominal}}{k}\right)^k$$

Question 1 (Gr 11/ November 2018)

1.1 Calculate the effective interest rate per annum if an investment earns interests at a rate of 11,5% p.a., compounded monthly.

1.2 Karabo bought a computer for R4 700. The value of the computer depreciated at a rate of 18% p.a. Using the reducing-balance method, calculate the book value of the computer 4 years after it was bought.

1.3 Nhlanhla made an initial deposit of R20 000 into an investment account that paid interest at the rate of 7,2% p.a., compounded quarterly. After 2 years the interest rate changed to 7,8% p.a., compounded monthly. Four years after his initial deposit, Nhlanhla withdrew R2 500 from his investment.

1.3.1 Calculate how much Nhlanhla had in this investment account 2 years after the initial deposit was made.

1.3.2 How much will the investment be worth 7 years after the initial deposit was made?

Question 2 (Gr 11/ November 2017)

2.1 A company bought machinery costing R80 000. Using the reducing balance method, the machinery had a book value of R20 000 after 5 years.
Calculate the rate of depreciation.

2.2 Calculate the effective interest rate if interest is compounded at 5% p.a., compounded quarterly.

2.3 Sipho invested R30 000 for 6 years. The investment earned interest at 12% p.a., compounded monthly for the first two years. There after the interest rate changed to 10,8% p.a., compounded semi-annually for the rest of the period.
Calculate the value of the investment at the end of 6 years. (No other transactions were made on the account.)

2.4 Mary deposited R25 000 into a savings account with an interest rate of 18% p.a., compounded monthly. Mary withdrew R8 000 from the account 2 years after depositing the initial amount. She deposited another R4 000 into this account 3 ½ years after the initial deposit. What amount will Mary have 5 years after making the initial deposit in this account?

Question 3 (Gr 11/ November 2016)

3.1 A machine costs R25 000 in 2016. Calculate the book value of the machine after 4 years if it depreciates at 9% p.a. according to the reducing balance method.

3.2 The nominal interest rate of an investment is 12,35% p.a., compounded monthly. Calculate the effective interest rate.

3.3 The value of a property increased from R145 000 to R221 292,32 over 6 years. Calculate the average annual rate of increase of the property over 6 years.

3.4 Tebogo made an initial deposit of R15 000 into an account that paid interest at 9,6% p.a., compounded quarterly. Six months later she withdrew R5 000 from the account. Two years after the initial deposit she deposited another R3 500 into this account. How much does she have in the account 3 years after her initial deposit?

Question 4 (Gr 11/ November 2015)

4.1 A school buys tablets at a total cost of R140 000. If the average rate of inflation is 6,1% per annum over the next 4 years, determine the cost of replacing these tablets in 4 years' time.

4.2 An investment earns interest at a rate of 7% per annum, compounded semi-annually. Calculate the effective annual interest rate on this investment.

4.3 A savings account was opened with an initial deposit of R24 000. Eighteen months later R7 000 was withdrawn from the account. Calculate how much money will be in the savings account at the end of 4 years if the interest rate was 10,5% p.a., compounded monthly.

4.4 A car costing R198 000 has a book value of R102 755,34 after 3 years. If the value of the car depreciates at r% p.a. on a reducing balance, calculate r.