Western Cape Government

Education

Directorate: Curriculum FET

2014 Telematics Grade 12

Mathematics

Price: R2,00

Session 1 & 2: Sequences and Series

Question 1

Consider the sequence:

$9 ; 4 ; -1 ; -6 ; ...$

1.1 Write down the next term of this sequence if the pattern continues. (1)

1.2 Calculate the sum of the first 20 terms of this sequence. (5)

1.3 Given the geometric sequence: $27 ; 9 ; 3 \ldots$

3.1.1 Determine a formula for T_n, the n^{th} term of the sequence. (2)

3.1.2 Why does the sum to infinity for this sequence exist? (1)

3.1.3 Determine S_∞. (2)

1.4 Given the geometric series: $5.(3)^4 + 5.(3)^3 + 5.(3)^2 + \ldots\ldots..$

1.4.1 Explain why the series converges. (2)

1.4.2 Calculate the sum to infinity of the series. (3)

1.4.3 Calculate the sum of the first 9 terms of the series, correct to **TWO** decimal places. (4)

1.4.4 Use your answers to QUESTION 1.4.2 and QUESTION 1.4.3 to determine $\sum_{n=10}^{\infty} 5.(3)^{5-n}$ (correct to **TWO** decimal places) (2)

1.5 Consider the following sequence: 6; 18; 54; 162 ……..

1.5.1 If the formula for the general term of the sequence is $T_n = ar^{n-1}$, give the value of a and r. (2)

1.5.2 Which term of the sequence is equal to 1458? (4)

QUESTION 2

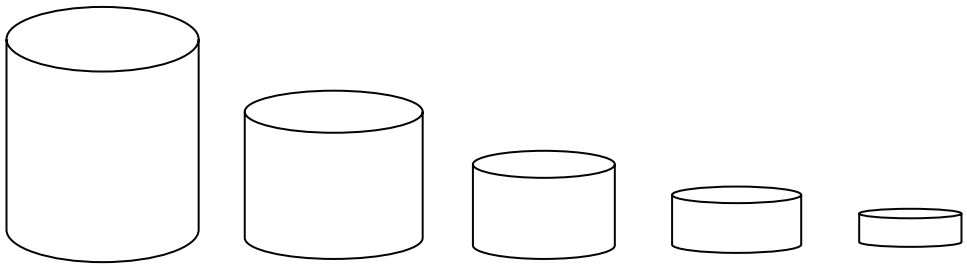
2.1 $3x + 1 ; 2x ; 3x - 7$ are the first three terms of an arithmetic sequence. Calculate the value of x. (2)

2.2 The first and second terms of an arithmetic sequence are 10 and 6 respectively.

2.2.1 Calculate the 11^{th} term of the sequence. (2)

2.2.2 The sum of the first n terms of this sequence is –560. Calculate n. (6)

3.2 Twenty water tanks are decreasing in size in such a way that the volume of each tank is $\frac{1}{2}$ the volume of the previous tank. The first tank is empty, but the other 19 tanks are full of water.



Would it be possible for the first water tank to hold all the water from the other 19 tanks? Motivate your answer. (4)

QUESTION 3

A Boeing 747 has been flying at a constant speed, but needs to decelerate before landing. In the last few minutes before landing, the distance it flies is 20km in the first minute, 16km in the second minute, 12,8km in the third minute, and so on, the numbers forming a geometric sequence.

5.1 Determine the common ratio. (1)

5.2 What will the n^{th} term be? (1)

5.3 Determine how far it will be travelling in the fourth minute. (2)

5.4 Write down in sigma notation, the total distance before landing. (1)

5.5 Determine the total distance it flies before landing. (2)

Session 3

Question 1

1.1 A business buys a machine that costs R120 000. The value of the machine depreciates at 9% per annum according to the diminishing-balance method.

1.1.1 Determine the scrap value of the machine at the end of 5 years. (3)

1.1.2 After five years the machine needs to be replaced. During this time, inflation remained constant at 7% per annum. Determine the cost of the new machine at the end of 5 years. (3)

1.1.3 The business estimates that it will need R90 000 by the end of five years. A sinking fund for R90 000, into which equal monthly instalments must be paid, is set up. Interest on this fund is 8,5% per annum, compounded monthly. The first payment will be made immediately and the last payment will be made at the end of the 5-year period. (5)
Calculate the value of the monthly payment into the sinking fund.

1.2

(6)

QUESTION 2

2.1 Sipho invests R5 000 at an interest rate of 8,4% per annum, compounded annually. After how many years will he be able to withdraw R8 793? (nearest year) (4)

2.2 Lorraine receives an amount of R900 000 upon her retirement. She invests this amount immediately at an interest rate of 10,5% per annum, compounded monthly. She needs an amount of R18 000 per month to maintain her current lifestyle. She plans to withdraw the first amount at the end of the first month. For how many months will she be able to live from her investment?

2.3 Peter takes out a loan of R560 000 to be able to buy a house. He pays back the loan over a period of 20 years, starting one month after he bought the house. Interest is charged at 11% p.a., compounded monthly.

2.3.1 Determine his monthly payments (correct to the nearest cent).

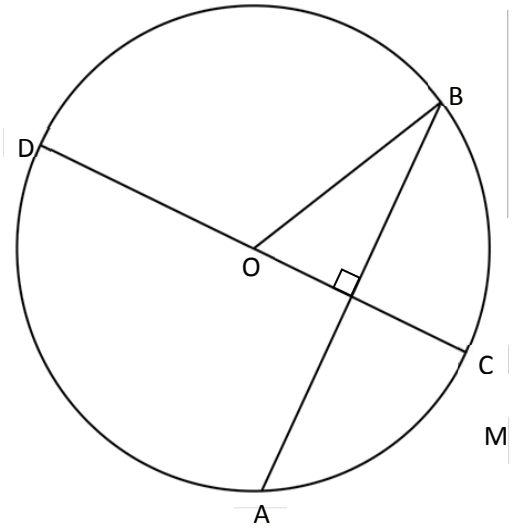
2.3.2 How long would it take to repay the loan if he were to pay R10 000 each month?

Session 4: Grade 11 geometry

Grade 11 theorems

1. The line drawn from the centre of a circle perpendicular to the chord bisects the chord
2. The perpendicular bisector of a chord passes through the centre of the circle
3. The angle subtended by an arc at the centre of a circle is double the angle subtended by the same arc at the circle (on the same side of the arc as the centre)
4. Angles subtended by an arc or chord of the circle on the same side of the chord are equal
5. The opposite angles of a cyclic quadrilateral are supplementary
6. Two tangents drawn to a circle from the same point outside the circle are equal in length (I**f two tangents to a circle are drawn from a point outside the circle, the distances between this point and the points of contact are equal)**.
7. The angle between the tangent of a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.

1. In the diagram below, O is the centre of the circle. Chord AB is perpendicular to diameter DC. CM : MD = 4 : 9 and AB = 24 units.



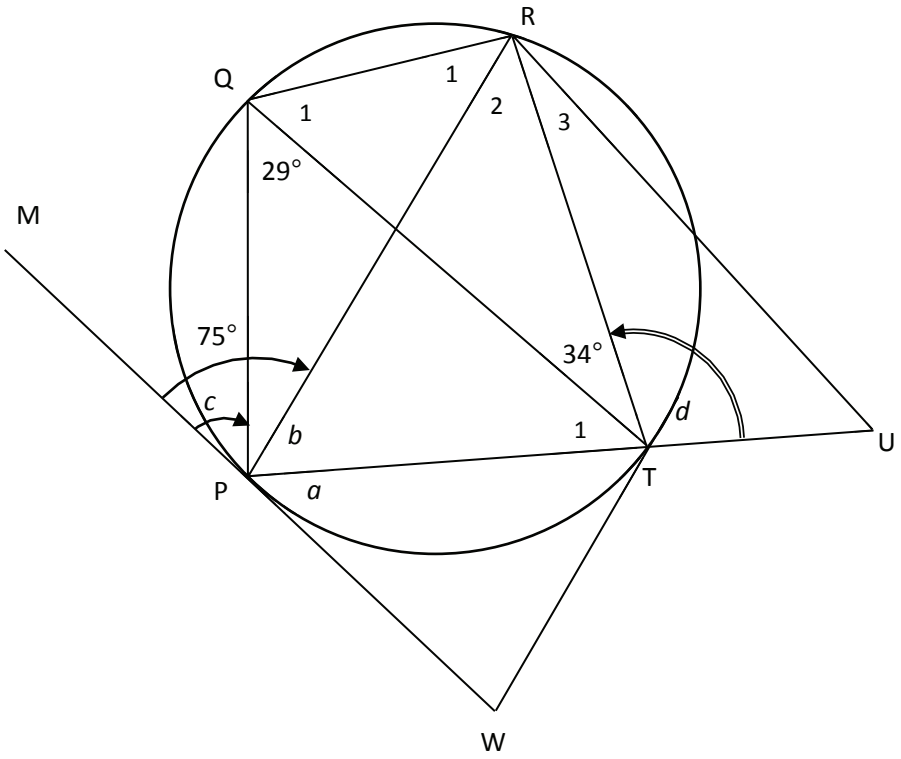
(a) Determine an expression for DC in terms of x if CM = $4x$ units.

(b) Determine an expression for OM in terms of x.

(c) Hence, or otherwise, calculate the length of the radius.

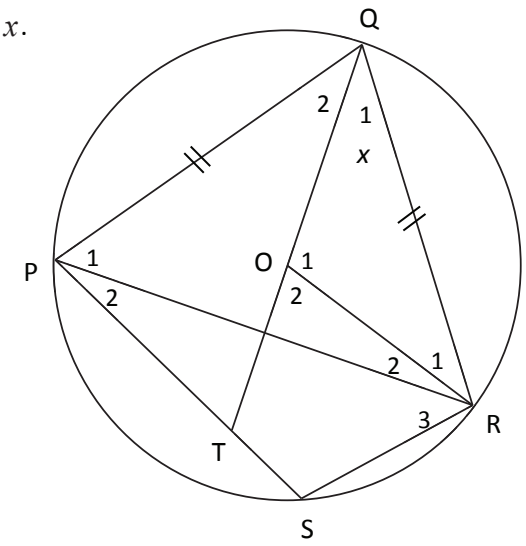
2 In the diagram points P, Q, R and T lie on the circumference of a circle. MW and TW are tangents to the circle at P and T respectively. PT is produced to meet RU at U. $\hat{MPR} = 75°$ $\hat{PQT} = 29°$ $\hat{QTR} = 34°$

Let $\hat{TPW} = a$, $\hat{RPT} = b$, $\hat{MPQ} = c$ and $\hat{RTU} = d$, calculate the values of a, b, c and d.



3 In the diagram below, O is the centre of the circle. P, Q, R and S are points on the circumference of the circle. TOQ is a straight line such that T lies on PS.

PQ = QR and $\hat{Q}_1 = x$.



3.1 Calculate, with reasons, $\hat{P}_1$ in terms of x. (3)

3.2 Show that TQ bisects $P\hat{Q}R$. (3)

3.3 Show that STOR is a cyclic quadrilateral. (3)

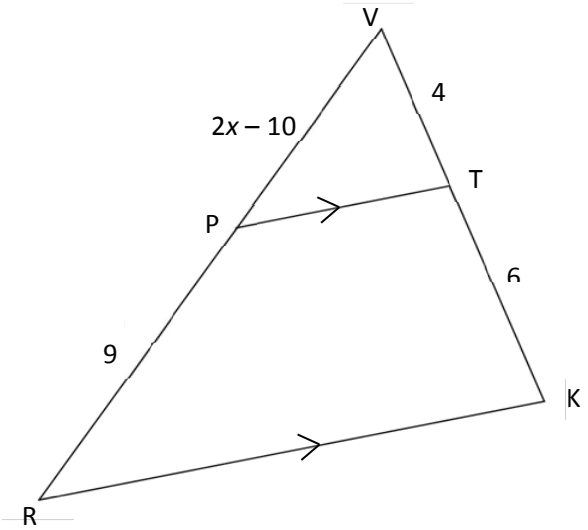
Session 5

Grade 12 theorems

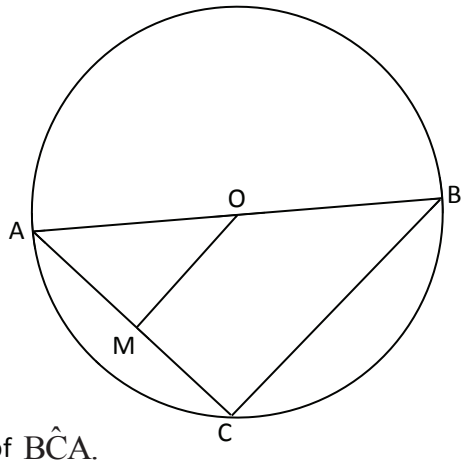
- The line drawn parallel to one side of a triangle divides the other two sides proportionally (The midpoint theorem is a special case of this theorem)
- Equiangular triangles are similar
- Triangles with sides in proportion are similar
- Prove the Pythagorean theorem by similar triangles

1 In the diagram below, ΔVRK has P on VR and T on VK such that PT || RK. VT = 4 units, PR = 9 units, TK = 6 units and VP = $2x - 10$ units.

Calculate the value of x.



2. O is the centre of the circle below. OM ⊥ AC. The radius of the circle is equal to 5 cm and BC = 8 cm.



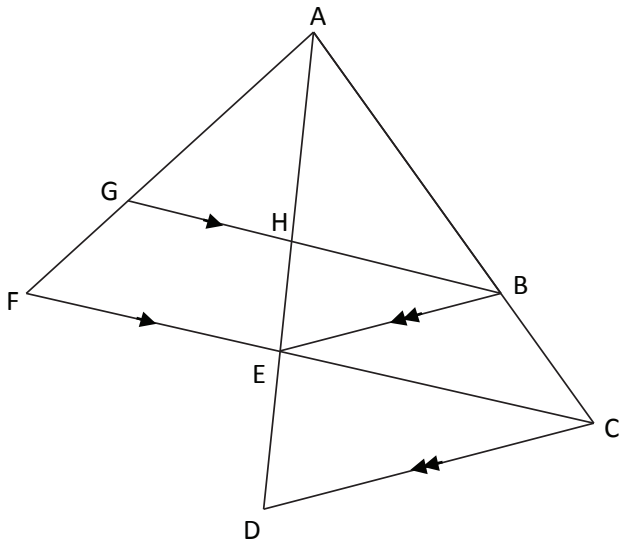
2.1 Write down the size of $B\hat{C}A$. (1)

2.2 Calculate:

2.2.1 The length of AM, with reasons (3)

2.2.2 Area ΔAOM : Area ΔABC (3)

In the figure below, GB || FC and BE || CD. AC = 6 cm and $\frac{AB}{BC} = 2$.



10.1 Calculate with reasons:

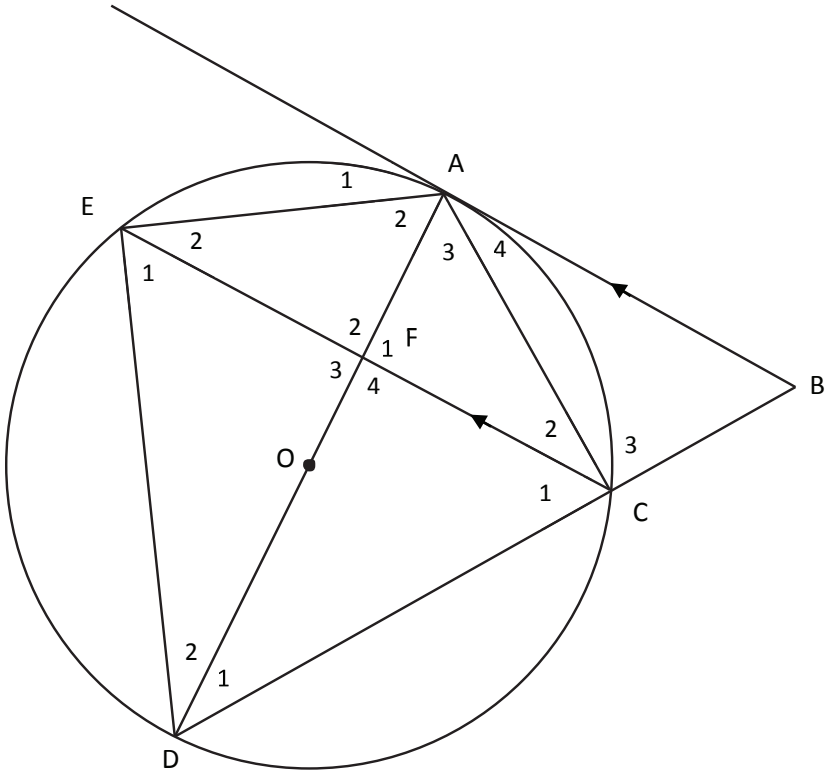
10.1.1 AH : ED (4)

10.1.2 $\frac{BE}{CD}$ (2)

10.2 If HE = 2 cm, calculate the value of AD × HE. (2)

[8]

3. In the figure below, AB is a tangent to the circle with centre O. AC = AO and BA || CE. DC produced, cuts tangent BA at B.

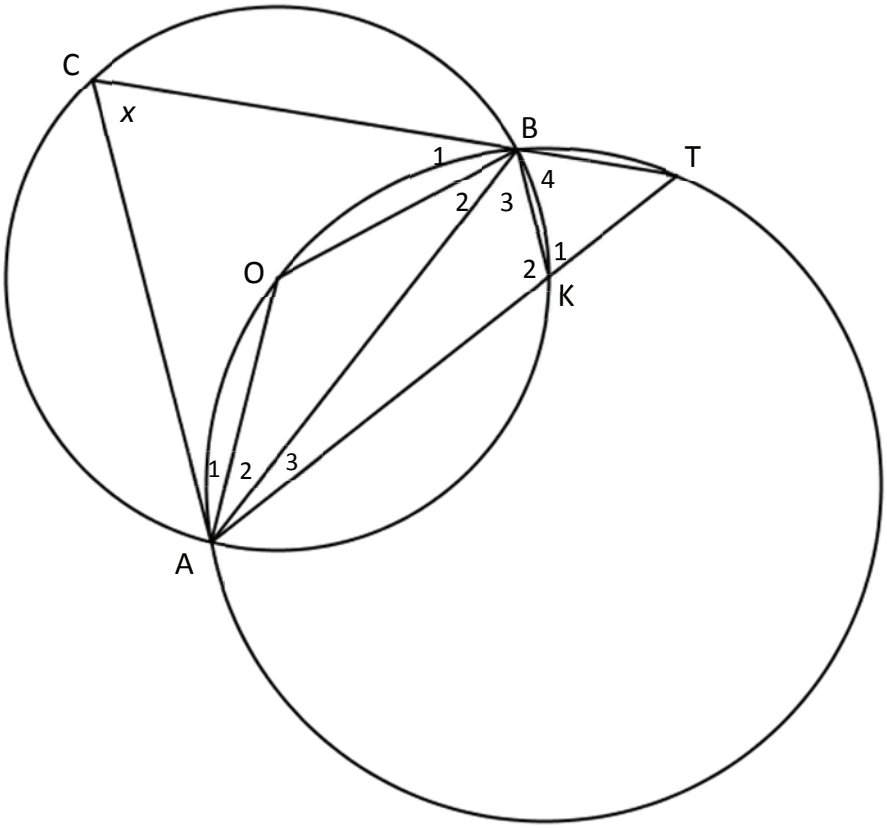


2.1 Show $\hat{C}_2 = \hat{D}_1$. (3)

2.2 Prove that $\Delta ACF \,|||\, \Delta ADC$. (3)

2.3 Prove that AD = 4AF. (4)

3. O is the centre of the circle CAKB. AK produced intersects circle AOBT at T. $A\hat{C}B = x$



3.1	Prove that $\hat{T} = 180° - 2x$.	(3)
3.2	Prove AC || KB.	(5)
3.3	Prove ΔBKT ||| ΔCAT	(3)
3.4	If AK : KT = 5 : 2, determine the value of $\frac{AC}{KB}$	(3)
		[14]

This learning materials is meant to support the 2014 cohort of Grade 12 mathematics learners from all the participating schools in the Telematics programme. This resource is meant for use alongside the broadcasts where the tutors will work through the exercises
The following topics will be handled in 2014:

Session 1 & 2: Revision of all the work Sequences and Series.
Session 3 Grade 12 financial mathematics, i.e. present and future value problems
Session 4 Revision of grade 11 geometry, i.e. circles and cyclic quadrilaterals and tangents
Session 5 Grade 12 geometry: Similar triangles and proportionality