Western Cape Government

Education

Western Cape Education Department

Telematics
Learning Resource 2018

MATHEMATICS
Grade 12

Dear Grade 12 Learner

In 2018 there will be 4 Telematics sessions on grade 12 content, and there are 3 sessions on grade 11 content. Please note that 60% of the content grade 12 learners are assessed on in the June, September and end of year examination is grade 11 content. As a grade 12 learner you could thus benefit from the grade 11 sessions as well. On page three the dates for the grade 12 and grade 11 sessions are given. This workbook provides the activities for the 4 sessions on grade 12 content. Please make sure that you bring this workbook along to each and every Telematics session.
In term one the presenters will revise Inverse Function and in term 2 the Log graph as the inverse of the Exponential graph. Please ensure that you revise all the graphs done in grade 11 before these sessions start. In the grade 12 examination Inverse Functions and graphs will be $\pm$ 35 marks of the 150 marks of Paper 1.

In term 3, Differential Calculus with specific focus on the Cubic graph is done in the 3th Mathematics Telematics Session. This will be followed by a Mathematics session for grade 11 learners which will be revision of Grade11 geometry. The Grade 11 geometry entails the circle geometry theorems dealing with angles in a circle, cyclic quadrilaterals and tangents. The grade 11 geometry material is not included in this workbook, it however is highly recommended that you attend this session, which will be followed by a session on Grade 12 geometry. Grade 12 geometry is based on ratio and proportion as well as similar triangles. Grade 11 geometry is especially important in order to do the grade 12 Geometry hence this work must be thoroughly understood and regularly practiced to acquire the necessary skills.

Your teacher should indicate to you exactly which theorems you have to study for examination purposes. There are altogether 6 proofs of theorems you must know because it could be examined. These theorems are also marked with (**) in this Telematics workbook. Four of the six theorems you have done in grade 11 and 2 are grade 12 theorems.

At the start of each lesson, the presenters will provide you with a summary of the important concepts and together with you will work though the activities. You are encouraged to come prepared, have a pen and enough paper (ideally a hard cover exercise book) and your scientific calculator with you.

You are also encouraged to participate fully in each lesson by asking questions and working out the exercises, and where you are asked to do so, sms or e-mail your answers to the studio.

Remember:" Success is not an event, it is the result of regular and consistent hard work".

GOODLUCK, Wishing you all the success you deserve!

Term 1: 17 Jan – 28 March					
Day	**Date**	**Time**	**Grade**	**Subject**	**Topic**
Monday	26 February	15:00 – 16:00	12	Mathematics	Inverse Functions

TERM 2: 10 April to 22 June					
Day	**Date**	**Time**	**Grade**	**Subject**	**Topic**
Tuesday	24 April	15:00 – 16:00	12	Mathematics	Graphs

Term 3: 17 July – 28 September					
Day	**Date**	**Time**	**Grade**	**Subject**	**Topic**
Tuesday	24 July	15:00 – 16:00	12	Mathematics	Calculus
Tuesday	31 July	16:00 – 17:00	11	Mathematics	Geometry
Monday	13 August	16:00 – 17:00	12	Mathematics	Geometry

Term 4: 9 October – 12 December					
Day	**Date**	**Time**	**Grade**	**Subject**	**Topic**
Monday	15 October	16:00 – 17:00	11	Mathematics	Trigonometry 1
Monday	22 October	16:00 – 17:00	11	Mathematics	Trigonometry 2

Session 1: The concept of an inverse; the inverses of $y = mx + c$ and $y = ax^2$

An inverse function is a function which does the "reverse" of a given function. More formally, if f is a function with domain *X*, then f^{-1} is its inverse function if and only if $f^{-1}(f(x)) = x$ for every $x \in X$.

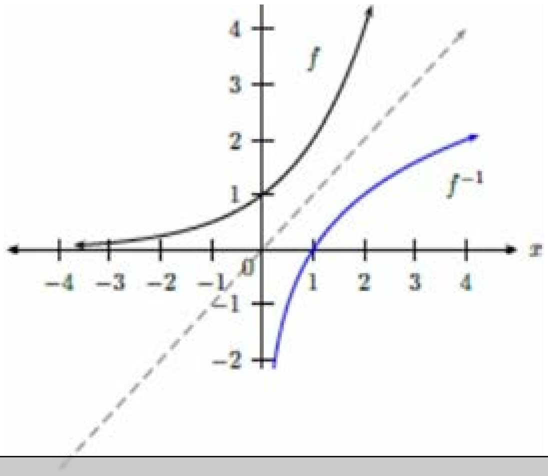
A function must be a one-to-one relation if its inverse is to be a function. If a function f has an inverse f^{-1}, then f is said to be invertible.

Given the function $f(x)$, we determine the inverse $f^{-1}(x)$ by:

- Interchanging *x* and *y* in an equation;
- Making *y* the subject of the equation;
- Expressing the new equation in function notation.

Note:
If the inverse is not a function then it cannot be written in function notation. For example, the inverse of $f(x) = 3x^2$ **cannot** be written as $f^{-1}(x) = \pm\sqrt{\frac{1}{3}x}$ as it is not a function. We write the inverse as $y = \pm\sqrt{\frac{1}{3}x}$ and conclude that $f(x) = 3x^2$ is not invertible.

If we represented the function f and the inverse f^{-1} graphically, the two graphs are reflected about the line $y = x$. Below is an example to illustrate this:



Important: for f^{-1}, the superscript -1 is not an exponent. It is the notation for indicating the inverse of a function. Do not confuse this with exponents, such as $(\frac{1}{2})^{-1}$ or $3 + x^{-1}$.

Be careful not to confuse the inverse of a function and the reciprocal of a function:

Inverse	**Reciprocal**
$f^{-1}(x)$	$\lvert f(x)\rvert^{-1} = \frac{1}{f(x)}$
$f(x)$ and $f^{-1}(x)$ are symmetrical about $y = x$	$f(x) \times \frac{1}{f(x)} = 1$
Example	Example
$g(x) = 5x \quad \therefore \quad g^{-1}(x) = \frac{x}{5}$	$g(x) = 5x \quad \therefore \quad \frac{1}{g(x)} = \frac{1}{5x}$

Example 1: An example of the inverse of $y = mx + c$.

Given $f(x) = 2x - 3$, draw $f(x)$ and $f^{-1}(x)$ on the same system of axes.

Given : $f(x) = 2x - 3$ $\quad \therefore \quad y = 2x - 3$

Step 1: Interchange *x* and *y*: $x = 2y - 3$

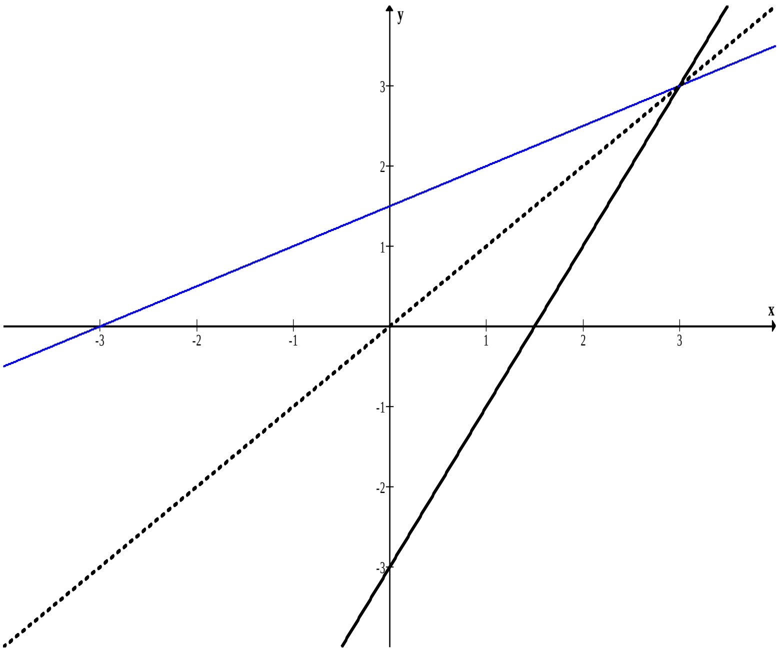
$$x + 3 = 2y$$

$$\frac{x}{2} + \frac{3}{2} = y$$

$$\therefore \quad y = \frac{x}{2} + \frac{3}{2}$$

Therefore, $f^{-1}(x) = \frac{x}{2} + \frac{3}{2}$

Step 2: Sketch the graphs on the same system of axes



The graph of $f^{-1}(x)$ is the reflection of $f(x)$ about the line $y = x$

Please note that when we are dealing with the inverse of a parabola (quadratic function), we encounter the problem that the inverse is not always a function. This is because the quadratic function is not a one-to-one relation (mapping). In order to ensure that we obtain a function for the inverse of the parabola, we must restrict the domain of the original function.

See the example below:

Example 2: An example of the inverse of $y = ax^2$.

Given $f(x) = 3x^2$, draw $f(x)$ and $f^{-1}(x)$ on the same system of axes.

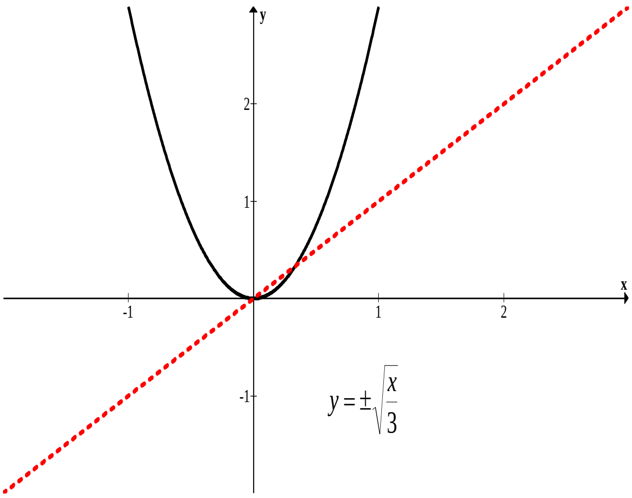
Given : $f(x) = 3x^2 \quad \therefore \quad y = 3x^2$

Step 1: **Interchange *x* and *y***: $x = 3y^2$

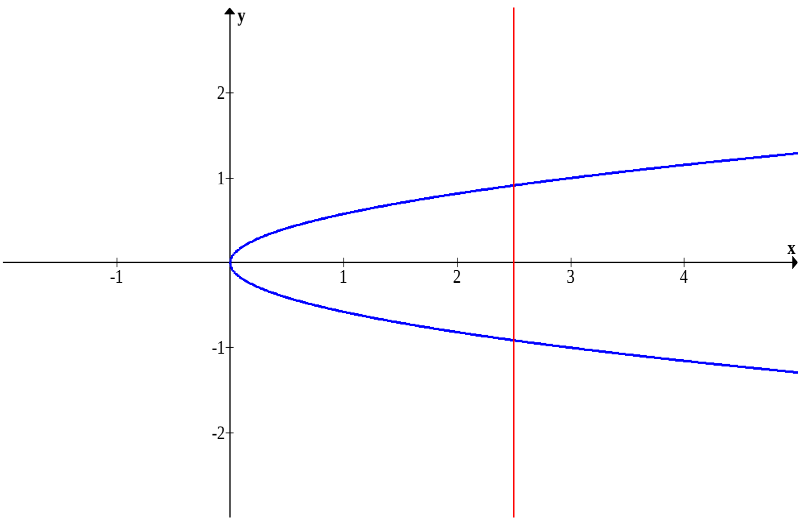
$$\frac{x}{3} = y^2$$

$$\therefore \quad y = \pm\sqrt{\frac{x}{3}} \quad \text{where} \quad x > 0$$

Step 2: Sketch the graphs on the same system of axes



Notice that the inverse does not pass the vertical line test and therefore is not a function.



To determine the inverse functions of $y = ax^2$:

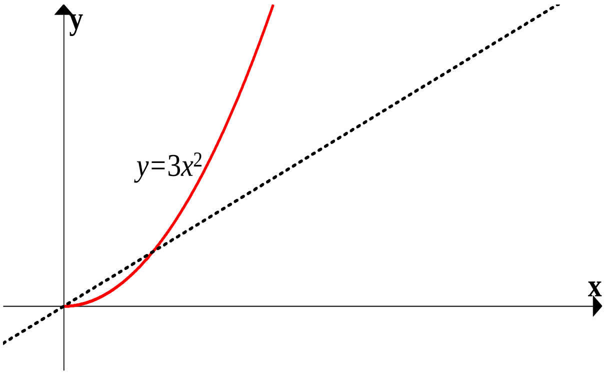
1) Interchange x and y: $x = ay^2$

2) Make y the subject of the equation: $\frac{x}{a} = y^2$

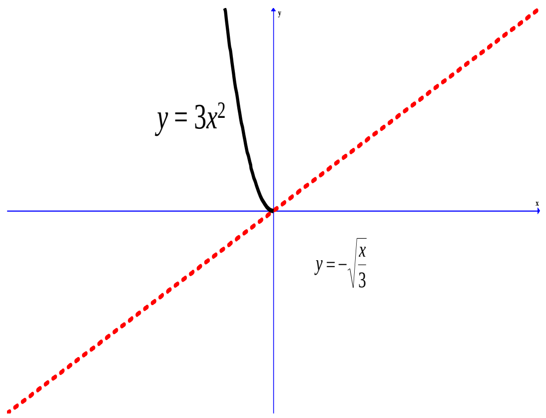
$y = \pm\sqrt{\frac{x}{a}}$ where $x \geq 0$

The vertical line test shows that the inverse of a parabola is not a function. However, we can limit the domain of the parabola so that the inverse of the parabola is a function. We can do this in two ways as illustrated below:

In the this sketch we have restricted the domain to $x \geq 0$.



In the sketch below we have restricted the domain to $x \leq 0$ then $f^{-1}(x) = -\sqrt{\frac{x}{3}}$ would also be a function.



Exercises:

The function concept

1. State if the following are true or false. Provide a reason for each answer.

   1.1 The inverse of $f = \{(2; 3); (4; 7)\}$ is $\{(3; 2); (7; 4)\}$ (2)

   1.2 $f = \{(2; -3); (4; 6); (-2; -3); (6; 4)\}$ is a many-to-one relation (2)

   1.3 The inverse of 1.2 is a function (2)

   1.4 The domain of 1.2 is D = {2; 4; 6} (2)

   1.5 The function f and its inverse f^{-1} are reflections in the line $y = -x$ (2)

The inverse of $y = mx + c$

2. Given $f(x) = 2x - 7$
   2.1 Is $f(x)$ a function? Explain your answer. (2)
   2.2 Write down the domain and range of $f(x)$ (2)
   2.3 Determine $f^{-1}(x)$ (2)
   2.4 Draw graphs of $f(x)$ and $f^{-1}(x)$ on the same system of axes. (4)
   2.5 Give the equation of the line of reflection between the two graphs and indicate this line on the graph using a broken line. (2)

3. Given that $f^{-1}(x) = -2x + 4$, determine $f(x)$. (2)

4. $f(x) = \frac{2}{3}x$ and $g(x) = -3x - 9$. Determine the point(s) of intersection of f^{-1} and g^{-1}. (7)

The inverse of $y = ax^2$

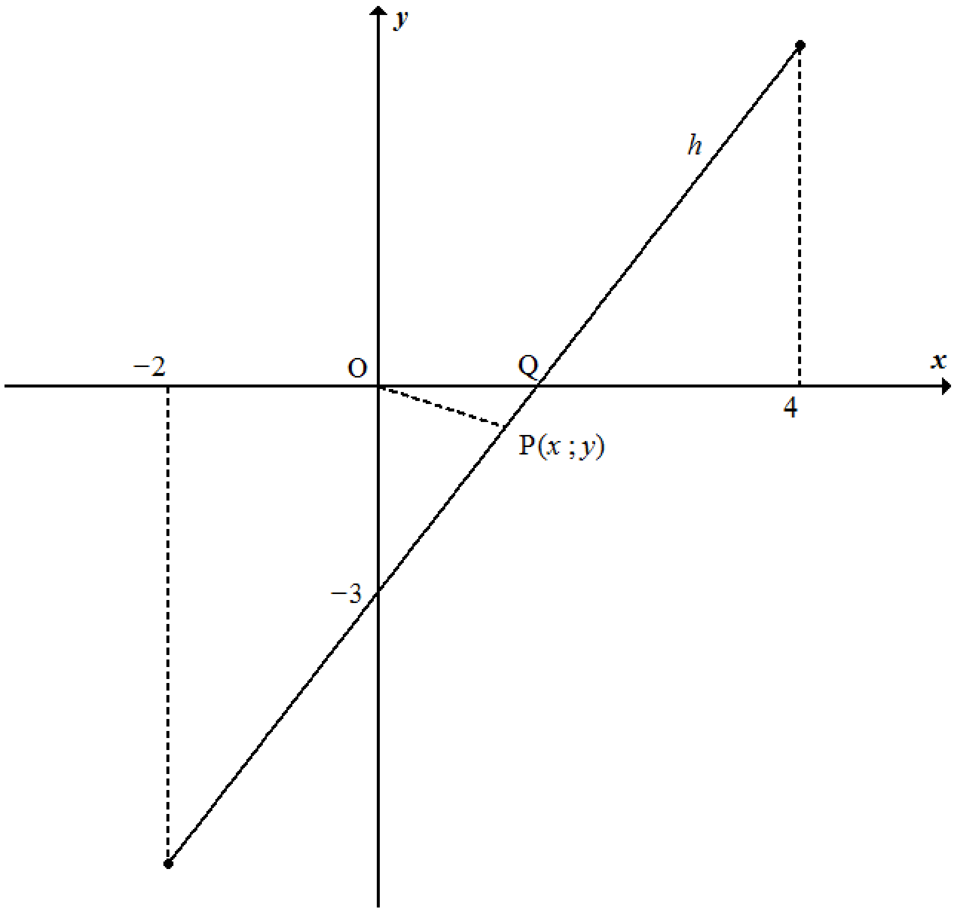
5. Given the function $f(x) = x^2$
   5.1 Determine $f^{-1}(x)$. (3)
   5.2 Draw the graph of $f^{-1}(x)$. (2)
   5.3 Explain why $f^{-1}(x)$ will not be a function? (1)
   5.4 Explain how you will restrict the domain of $f(x)$ to ensure that $f^{-1}(x)$ will also be a function. (2)

6. Given $f(x) = \frac{1}{2}x^2$
   6.1 Determine the inverse of $f(x)$ (3)
   6.2 Is the inverse of $f(x)$ a function or not? Give a reason for your answer. (2)
   6.3 How will you restrict the domain of the original function so as to ensure that $f^{-1}(x)$ will also be a function. (1)
   6.4 Draw graphs of $f(x)$ and $f^{-1}(x)$ on the same system of axes. (3)
   6.5 Determine the point(s) where $f(x)$ and $f^{-1}(x)$ will intersect each other. (4)

7. Given $f(x) = -2x^2$
   7.1 Explain why, if the domain of this function is not restricted, its inverse will not be a function? (2)
   7.2 Write down the equation of the inverse, $f^{-1}(x)$ of $f(x) = -2x^2$ for $x \in (-\infty ; 0]$ in the form $f^{-1}(x) = ...$ (3)
   7.3 Write down the domain of $f^{-1}(x)$. (2)
   7.4 Draw graphs of both $f(x) = -2x^2$ for $x \in (-\infty ; 0]$ and $f^{-1}(x)$ on the same system of axes. (4)

8.

Given: $h(x) = 2x - 3$ for $-2 \leq x \leq 4$. The x-intercept of h is Q.



8.1	Determine the coordinates of Q.	(2)
8.2	Write down the domain of h^{-1}.	(3)
8.3	Sketch the graph of h^{-1} in your ANSWER BOOK, clearly indicating the y-intercept and the end points.	(3)
8.4	For which value(s) of x will $h(x) = h^{-1}(x)$?	(3)
8.5	P(x ; y) is the point on the graph of h that is closest to the origin. Calculate the distance OP.	(5)
8.6	Given: $h(x) = f'(x)$ where f is a function defined for $-2 \leq x \leq 4$.	

8.6.1	Explain why f has a local minimum.	(2)
8.6.2	Write down the value of the maximum gradient of the tangent to the graph of f.	(1)

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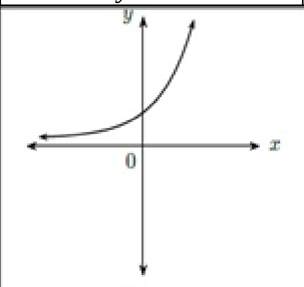
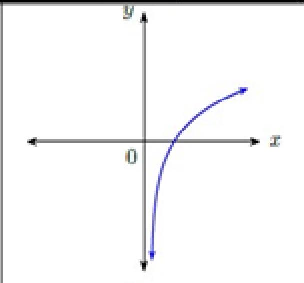
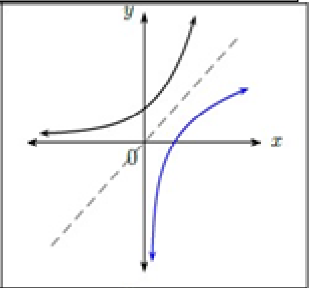
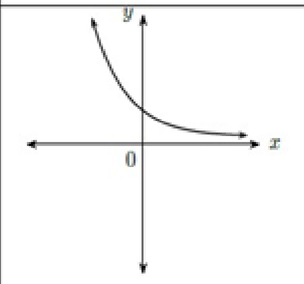
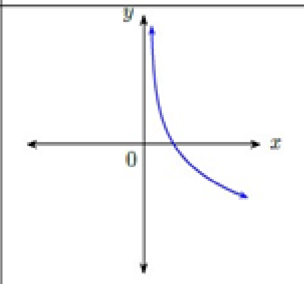
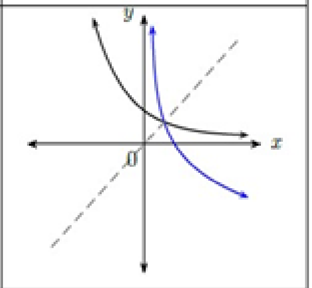
Session 2

The Log-function and its inverse:

Summary of graphs: $y = b^x$ **and** $y = \log_b x$

	Exponential function $y = b^x$	Logarithmic function $y = \log_b x$	Exponential & Logarithmic function
$b > 1$			
$0 < b < 1$			

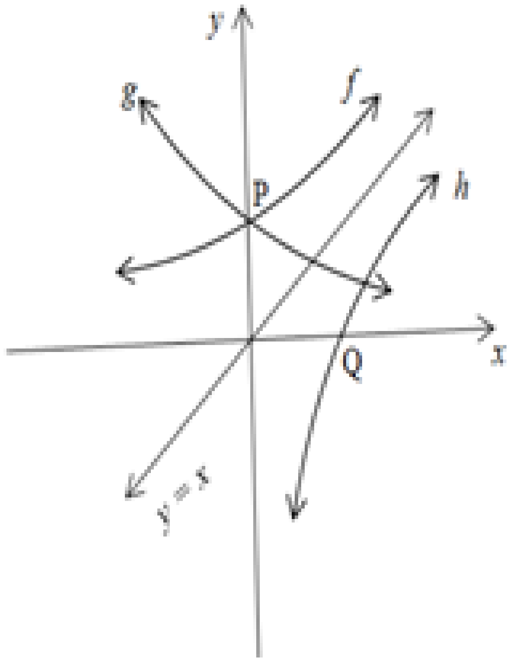
Log and exponential functions as inverses of each other

1. The graph alongside shows the functions *g*, *f* and *h*. *f* and *g* are symmetrical with respect to the *y*-axis. *f* and *h* are symmetrical with respect to the line $y = x$. If $f(x) = a^x$ and the point (1; 4) lies on $f(x)$:

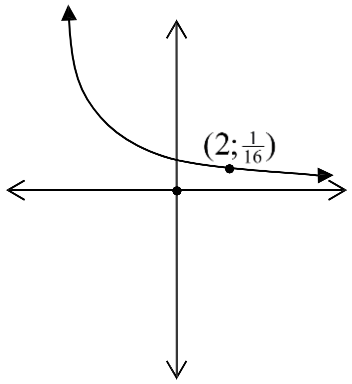
   1.1. Determine the value the value *a*. (2)

   1.2. Write down the coordinates of P and Q. (2)

   1.3. Write down the equation of *g*, *h*. and g^{-1}. (6)



2. The figure represents the graph of $f(x) = a^x$.



2.1 Calculate the value of a. (2)

2.2 Draw a graph of $k(x)$ if k is the inverse of f. Show the intercepts with the axes, as well as the coordinates of one other point. Also indicate the asymptotes. (4)

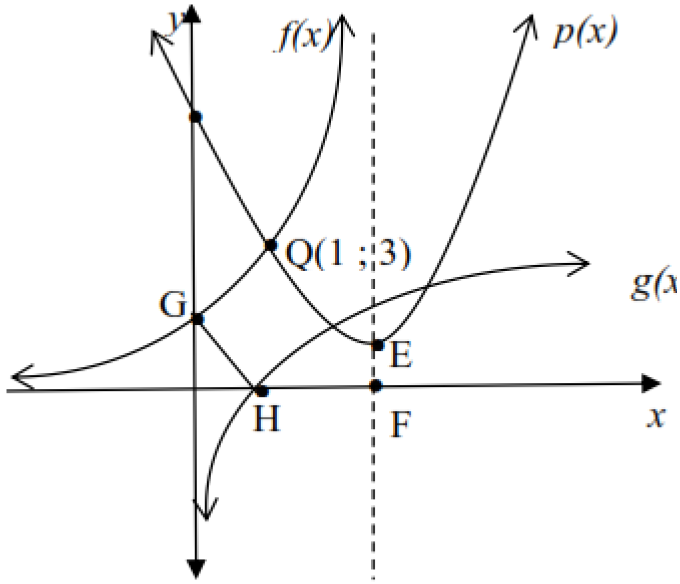
3. The diagram alongside show the functions:

$f(x) = k^x$;

$p(x) = ax^2 + bx + c$ and

$g(x) = \log_m x$

f and g are inverses of each other. The minimum value of the function $p(x)$ is equal to 1 where $x = 3$. The turning point of the parabola is at E. EF is parallel to the y-axis.



3.1 Determine the values of a, b, c, m and k. (5)

3.2 Calculate the length of EF and GH correct to two decimal places. (3)

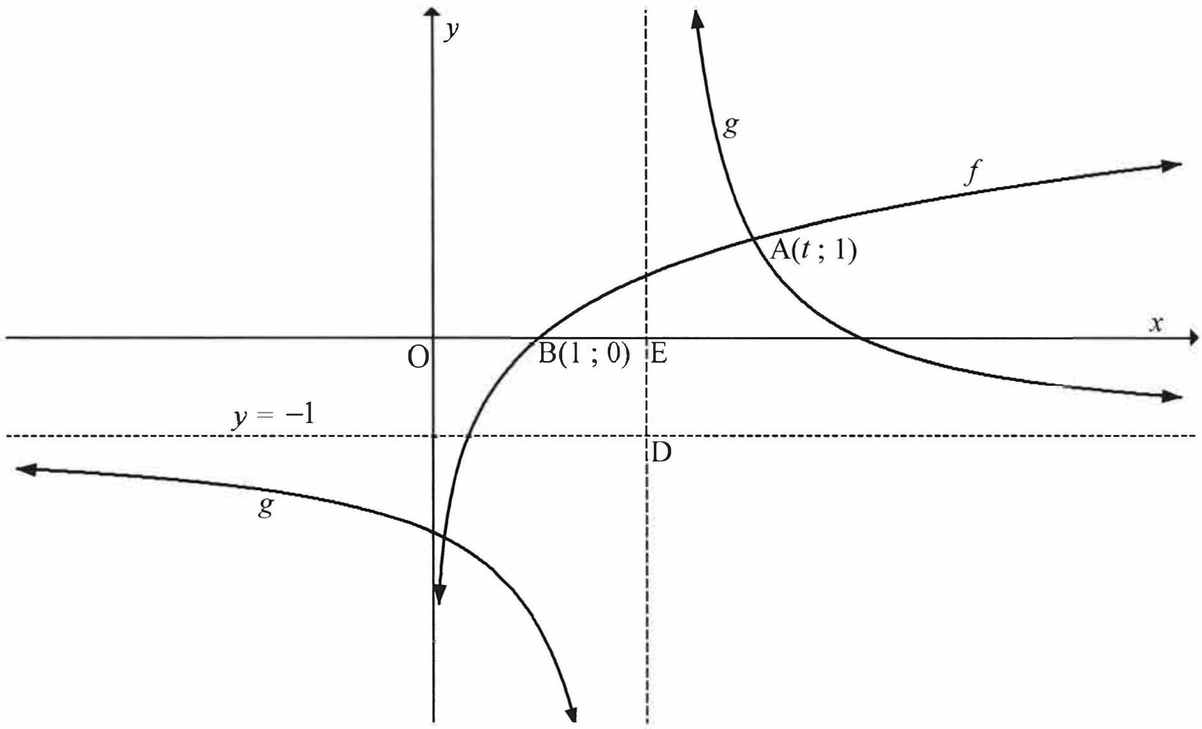
3.3 Determine the equation of $f^{-1}(x)$ and $g^{-1}(x)$. (4)

3.4 Hence explain why or why not, $f(x)$ and $g(x)$ will be symmetrical with respect to the line $y = x$. (1)

4.

The diagram below shows the graphs of $g(x)=\dfrac{2}{x+p}+q$ and $f(x)=\log_3 x$.

- $y=-1$ is the horizontal asymptote of g.
- B(1 ; 0) is the x-intercept of f.
- A(t ; 1) is a point of intersection between f and g.
- The vertical asymptote of g intersects the x-axis at E and the horizontal asymptote at D.
- OB = BE.



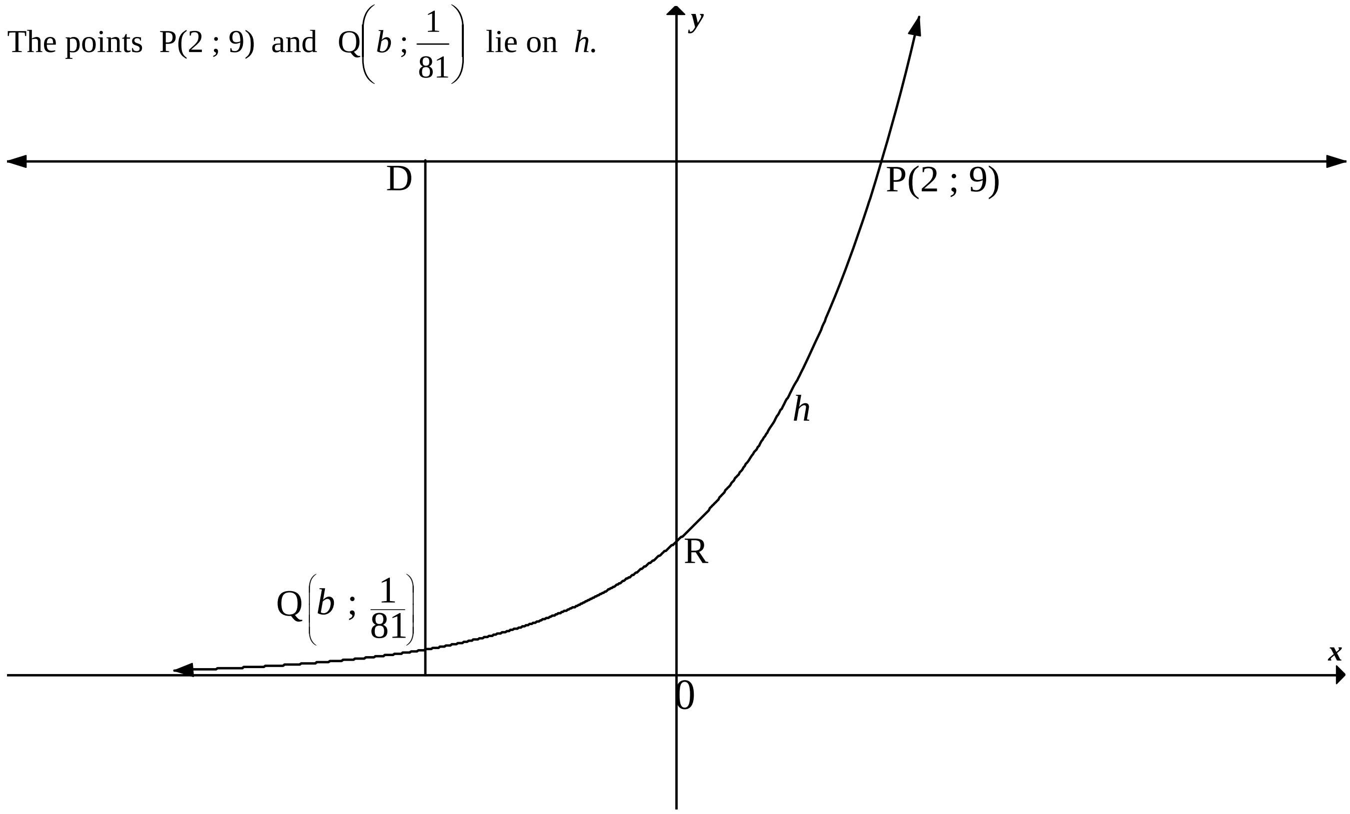
4.1	Write down the range of g.	(2)
4.2	Determine the equation of g.	(2)
4.3	Calculate the value of t.	(3)
4.4	Write down the equation of f^{-1}, the inverse of f, in the form $y=\ldots$	(2)
4.5	For which values of x will $f^{-1}(x)<3$?	(2)
4.6	Determine the point of intersection of the graphs of f and the axis of symmetry of g that has a negative gradient.	(3)

[14]

5.

Sketched below is the graph of $h(x) = a^x$, $a > 0$. R is the y-intercept of h.

The points P(2 ; 9) and $Q\left(b\ ;\ \frac{1}{81}\right)$ lie on h.



5.1	Write down the equation of the asymptote of h.	(1)
5.2	Determine the coordinates of R.	(1)
5.3	Calculate the value of a.	(2)
5.4	D is a point such that DQ ‖ y-axis and DP ‖ x-axis. Calculate the length of DP.	(4)
5.5	Determine the values of k for which the equation $h(x+2)+k=0$ will have a root that is less than –6.	(3)
		[11]

6.

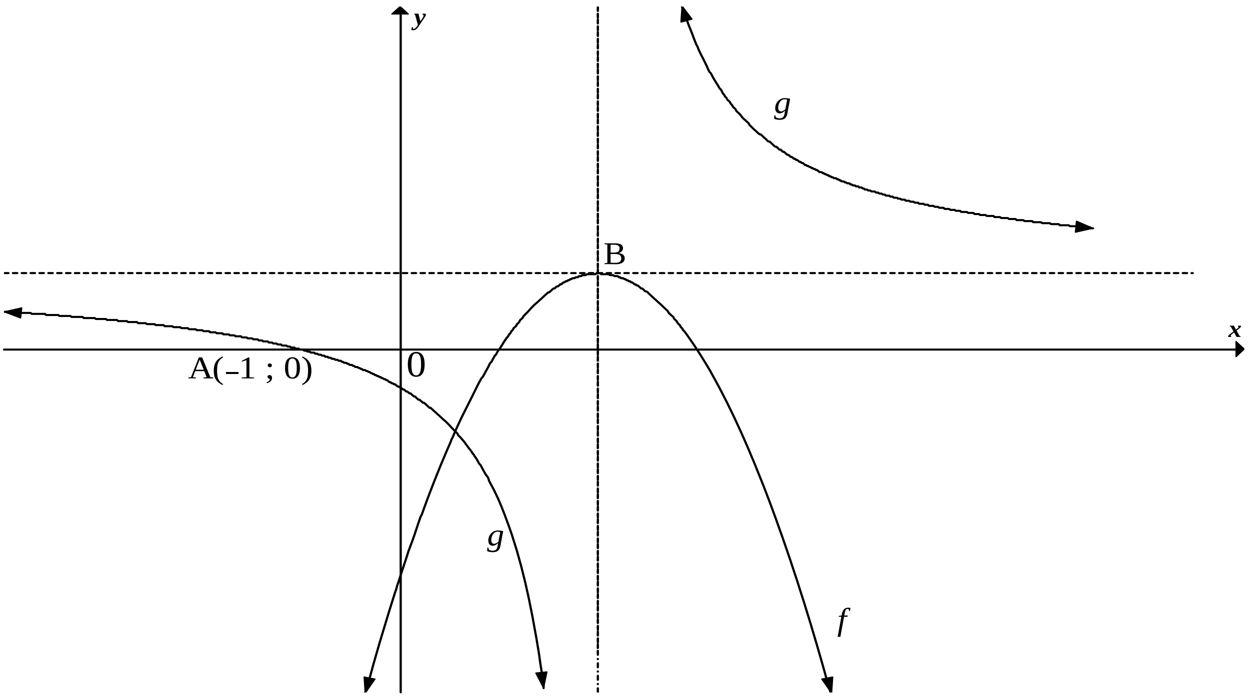
Given: $f(x) = -x + 3$ and $g(x) = \log_2 x$

6.1	On the same set of axes, sketch the graphs of f and g, clearly showing ALL intercepts with the axes.	(4)
6.2	Write down the equation of $g^{-1}(x)$, the inverse of g, in the form $y = \ldots$	(2)
6.3	Explain how you will use QUESTION 6.1 and/or QUESTION 6.2 to solve the equation $\log_2(3-x) = x$.	(3)
6.4	Write down the solution to $\log_2(3-x) = x$.	(1)
		[10]

QUESTION 7

Sketched below is the parabola f, with equation $f\left(x - = \right)x^2 + 4x - 3$ and a hyperbola g, with equation $(x - p)(y + t) = 3$.

- B, the turning point of f, lies at the point of intersection of the asymptotes of g.
- $A(-1\,;\,0)$ is the x-intercept of g.



7.1	Show that the coordinates of B are $(2\,;\,1)$	(2)
7.2	Write down the range of f.	(1)
7.3	For which value(s) of x will $g(x) \geq 0$?	(2)
7.4	Determine the equation of the vertical asymptote of the graph of h if $h(x) = g(x+4)$	(1)
7.5	Determine the values of p and t.	(4)
7.6	Write down the values of x for which $f(x).g^{/}(x) \geq 0$	(4)
		[14]

Session 3: Grade 12 Calculus

Cubic Graphs. In this lesson you will work through 3 types of questions regarding graphs

1. Drawing cubic graphs

2. Given the graphs, answer interpretive questions

3. Given the graphs of the derivative, answer interpretive questions

1.1

Given: $f(x) = 2x^3 - x^2 - 4x + 3$

1.1.1 Show that $(x-1)$ is a factor of $f(x)$. (2)

1.1.2 Hence factorise $f(x)$ completely. (2)

1.1.3 Determine the co-ordinates of the turning points of f. (4)

1.1.4 Draw a neat sketch graph of f indicating the co-ordinates of the turning points as well as the x-intercepts. (4)

1.1.5 For which value of x will f have a point of inflection? (4)

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1.2

Given $f(x) = x^3 + x^2 - 5x + 3$

1.2.1 Show that $(x-1)$ is a factor of $f(x)$. (2)

1.2.2 Factorise $f(x)$ fully. (3)

1.2.3 Determine the x and y intercepts of $f(x)$. (2)

1.2.4 Determine the co-ordinates of the turning point(s) of $f(x)$. (4)

1.2.5 Find the x-coordinate of the point of inflection of $f(x)$. (1)

1.2.6 Draw a sketch graph of $f(x)$. (2)

1.2.7 For which value(s) of x is $f(x)$ increasing? (2)

1.2.8 Describe one transformation of $f(x)$ that, when applied, will result in $f(x)$ having two unequal positive real roots. (2)

1.2.9 Give the equation of g if g is the reflection of f in the y-axis. (3)

1.2.10 Determine the average rate of change of f between the points (0 ; 3) and (1;0). (2)

1.2.11 Determine the equation of the tangent to f when x = -2. (4)

1.2.12 Prove that the tangent in 1.2.11 will intersect or touch the curve of f at two places. (4)

1.4

A cubic function f has the following properties:

- $f\left(\frac{1}{2}\right) = f(3) = f(-1) = 0$
- $f'(2) = f'\left(-\frac{1}{3}\right) = 0$

f decreases for $x \in \left[-\frac{1}{3}\,;\,2\right]$ only

Draw a possible sketch graph of f, clearly indicating the x-coordinates of the turning points and ALL the x-intercepts. (4)

1.3

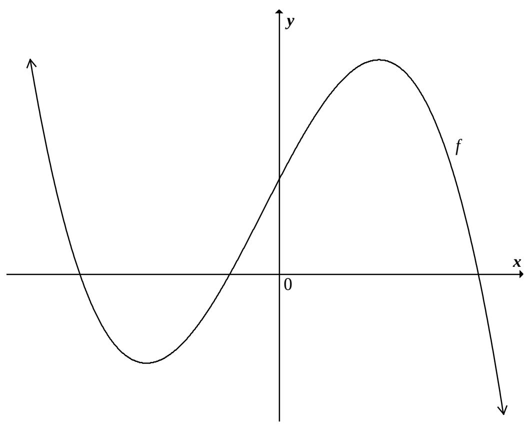
The tangent to the curve of $g(x) = 2x^3 + px^2 + qx - 7$ at $x = 1$ has the equation $y = 5x - 8$.

1.3.1 Show that $(1\,;-3)$ is the point of contact of the tangent to the graph. (1)

1.3.2 Hence or otherwise, calculate the values of p and q. (6)

2.1

The graph of the function $f(x) = -x^3 - x^2 + 16x + 16$ is sketched below.

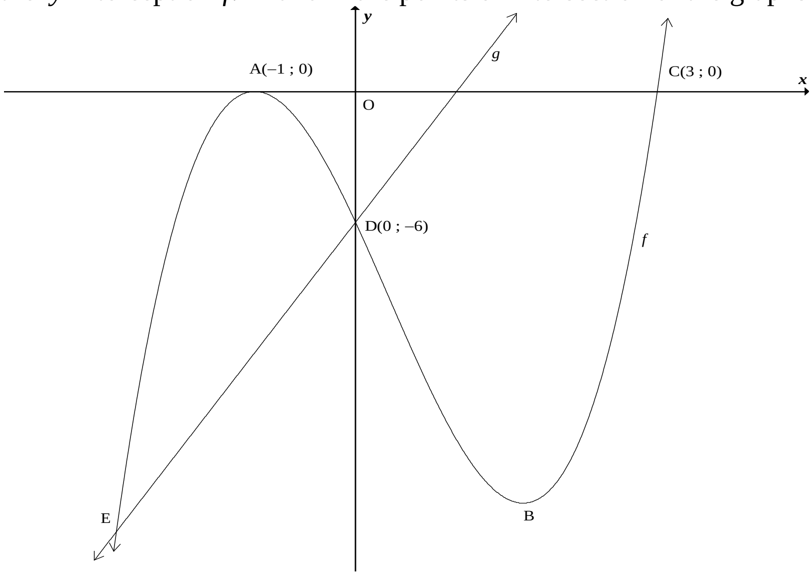


2.1.1 Calculate the x-coordinates of the turning points of f. (4)

2.1.2 Calculate the x-coordinate of the point at which $f'(x)$ is a maximum. (3)

2.2

The graphs of $f(x) = ax^3 + bx^2 + cx + d$ and $g(x) = 6x - 6$ are sketched below. A(– 1 ; 0) and C(3 ; 0) are the x-intercepts of f. .The graph of f has turning points at A and B. D(0 ; – 6) is the y-intercept of f. E and D are points of intersection of the graphs of f and g.



2.2.1 Show that $a = 2$; $b = -2$; $c = -10$ and $d = -6$. (5)

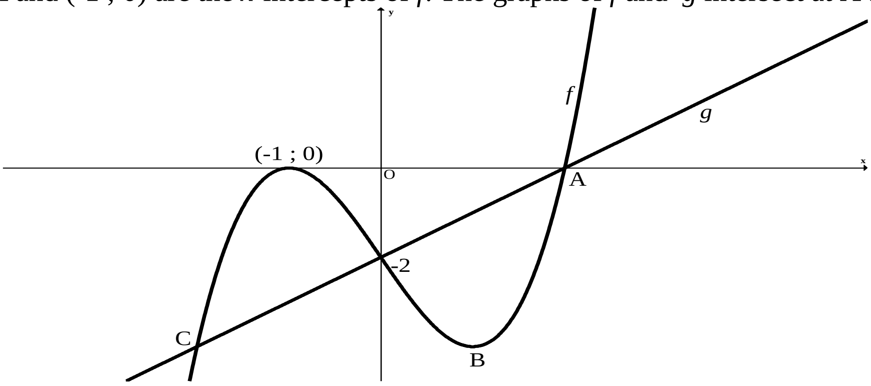
2.2.2 Calculate the coordinates of the turning point B. (5)

2.2.3 $h(x)$ is the vertical distance between $f(x)$ and $g(x)$, that is $h(x) = f(x) - g(x)$. (5)

Calculate x such that $h(x)$ is a maximum, where $x < 0$.. [15]

2.3

The graph below represents the function f and g with $f(x) = ax^3 - cx - 2$ and $g(x) = x - 2$. A and (-1 ; 0) are the x-intercepts of f. The graphs of f and g intersect at A and C.



2.3.1 Determine the coordinates of A. (1)

2.3.2 Show by calculations that $a = 1$ and $c = -1$. (4)

2.3.3 Determine the coordinates of B, a turning point of f. (3)

2.3.4 Show that the line BC is parallel to the x-axis. (7)

2.3.5 Find the x-coordinate of the point of inflection of f. (2)

2.3.6 Write down the values of k for which $f(x) = k$ will have only ONE root. (3)

2.3.7 Write down the values of x for which $f'(x) < 0$. (2)

2.3.8 Write down the values of x for which $f(x).g(x) < 0$.

2.4 Consider the graph of $g(x) = -2x^2 - 9x + 5$.

2.5.1 Determine the equation of the tangent to the graph of g at $x = -1$. (4)

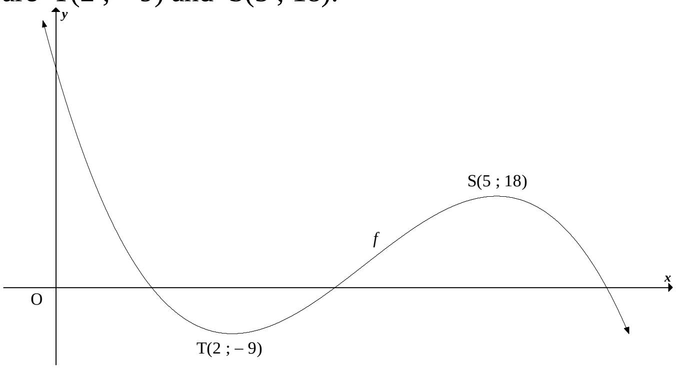
2.5.2 For which values of q will the line $y = -5x + q$ not intersect the parabola? (3)

2.5 Given: $h(x) = 4x^3 + 5x$

Explain if it is possible to draw a tangent to the graph of h that has a negative gradient. Show ALL your calculations. (3)

2.6

The function $f(x) = -2x^3 + ax^2 + bx + c$ is sketched below. The turning points of the graph of f are T(2 ; – 9) and S(5 ; 18).



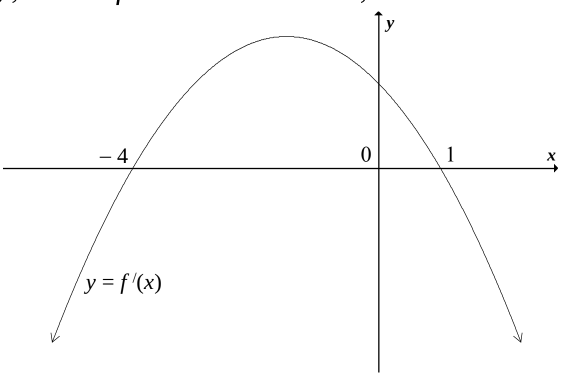
2.6.1 Show that $a = 21$, $b = -60$ and $c = 43$. (7)

2.6.2 Determine an equation of the tangent to the graph of f at $x = 1$. (5)

2.6.3 Determine the x-value at which the graph of f has a point of inflection. (2)

3.1

The graph of $y = f'(x)$, where f is a cubic function, is sketched below.



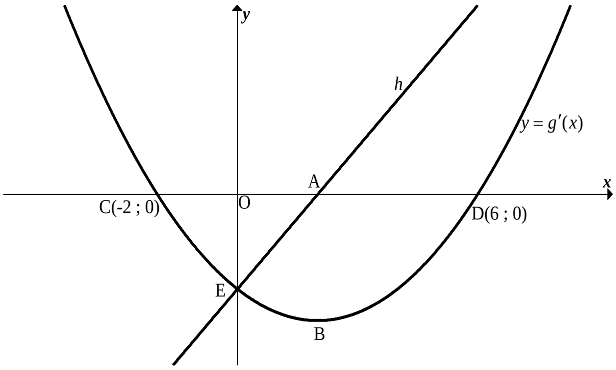
Use the graph to answer the following questions:

3.1.1 For which values of x is the graph of $y = f'(x)$ decreasing? (1)

3.1.2 At which value of x does the graph of f have a local minimum? Give reasons for your answer. (3)

3.2

The graphs of $y = g'(x) = ax^2 + bx + c$ and $h(x) = 2x - 4$ are sketched below. The graph of $y = g'(x) = ax^2 + bx + c$ is a derivative graph of a cubic function g. The graphs of h and g' have a common y-intercept at E. C(-2 ; 0) and D(6 ; 0) are the x-intercept of the graph of g'. A is the x-intercept of h and B is the turning point of g'. AB $\parallel$ y-axis.



3.2.1 Write down the coordinates of E. (1)

3.2.2 Determine the equation of the graph of g' in the form $y = ax^2 + bx + c$. (4)

3.2.3 Write down the x-coordinates of the turning points of g. (2)

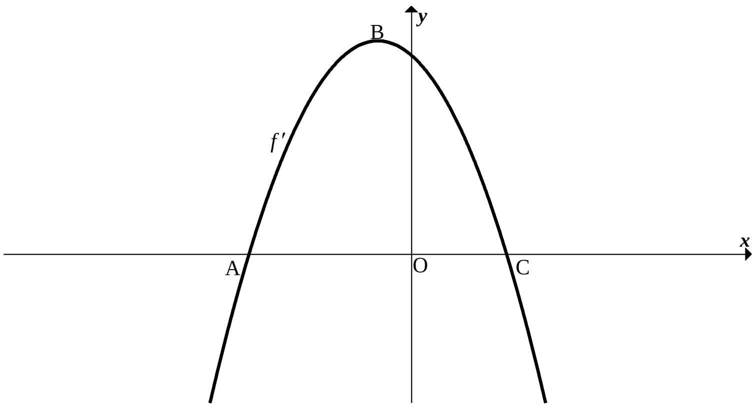
3.2.4 Write down the x-coordinates of the point of inflection of the graph of g. (2)

3.2.5 Explain why g has a local maximum at $x = -2$ (3)

3.2.6 Write down the values of x for which $h(x).g'(x) < 0$.

3.3

Sketched below is the graph of f', the derivative of $f(x) = -2x^3 - 3x^2 + 12x + 20$. A, B and C are the intercepts of f' with the axes.



3.3.1 Determine the coordinates of A. (2)

3.2.2 Determine the coordinates of B and C. (3)

3.2.3 Which points on the graph of $f(x)$ will have exactly the same x-coordinates as B and C? (1)

3.2.4 For which values of x will $f(x)$ be increasing? (2)

3.2.5 Determine the y-coordinates of the point of inflection of f. (4)

4

Given: $f(x) = x(x-3)^2$ with $f'(1) = f'(3) = 0$ and $f(1) = 4$

4.1 Show that f has a point of inflection at $x = 2$. (5)

4.2 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (4)

4.3 For which values of x will $y = -f(x)$ be concave down? (2)

4.4 Use your graph to answer the following questions:

4.4.1 Determine the coordinates of the local maximum of h if $h(x) = f(x-2)+3$. (2)

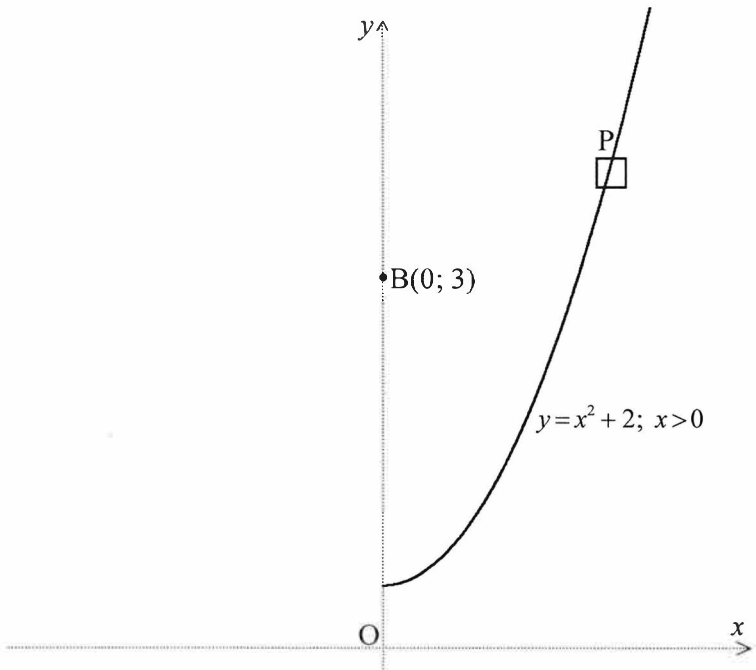
4.4.2 Claire claims that $f'(2) = 1$.

Do you agree with Claire? Justify your answer. (2)
[15]

5

An aerial view of a stretch of road is shown in the diagram below. The road can be described by the function $y = x^2 + 2$, $x \geq 0$ if the coordinate axes (dotted lines) are chosen as shown in the diagram.

Benny sits at a vantage point B(0 ; 3) and observes a car, P, travelling along the road.



Calculate the distance between Benny and the car, when the car is closest to Benny. **[7]**

Session 4
Grade 12 Geometry

Below are **Grade 12 Theorems**, Converse Theorems and their Corollaries which you must know. The proofs of the theorems marked with (**) must be studied because it could be examined.

1	**Theorem****	A line drawn parallel to one side of a triangle divides the other two sides proportionally. **(line || one side of Δ OR prop theorem; name || lines)**
	Converse	If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side. **(line divides two sides of Δ in prop)**
	Theorem**	If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar) **(||| Δs OR equiangular Δs)**
	Converse	If the corresponding sides of two triangles are proportional, then the triangles are equiangular (and consequently the triangles are similar). **(Sides of Δ in prop)**

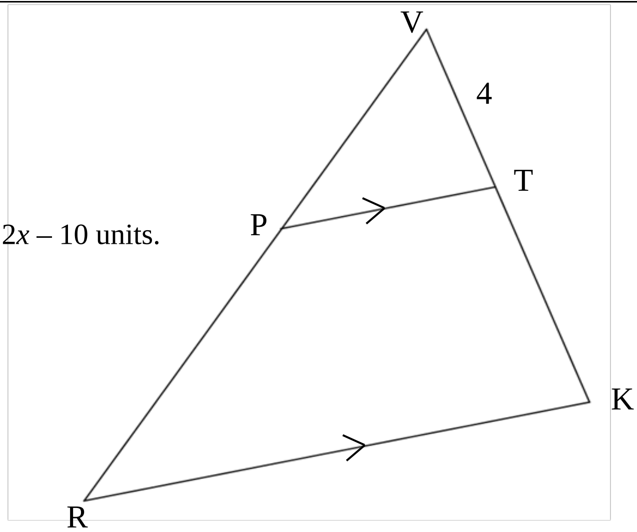
Question 1

1.1

In the diagram ΔVRK has P on VR and such that PT || RK.

VT = 4 units, PR = 9 units, TK = 6 units and VP = $2x - 10$ units.

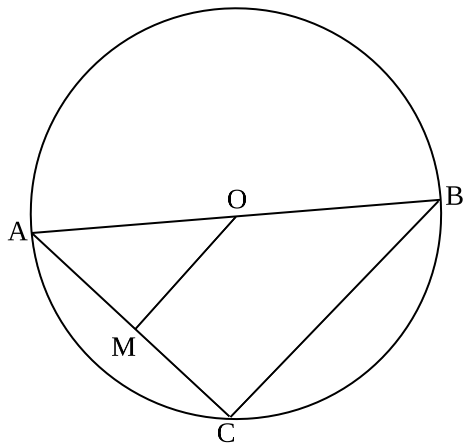
Calculate the value of x.



1.2

O is the centre of the circle below. OM ⊥ AC. The radius of the circle is equal to 5 cm and

BC = 8 cm.



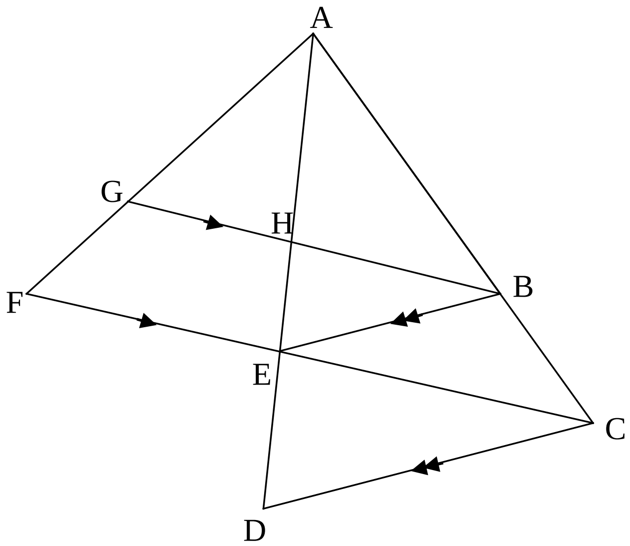
1.2.1 Write down the size of $B\hat{C}A$. (1)

1.2.2 Calculate:

(a) The length of AM, with reasons (3)

(b) Area ΔAOM : Area ΔABC (3)

1.3 In the figure below, GB || FC and BE || CD. AC = 6 cm and $\frac{AB}{BC} = 2$.



1.3.1 Calculate with reasons:

AH : ED (4)

$\frac{BE}{CD}$ (2)

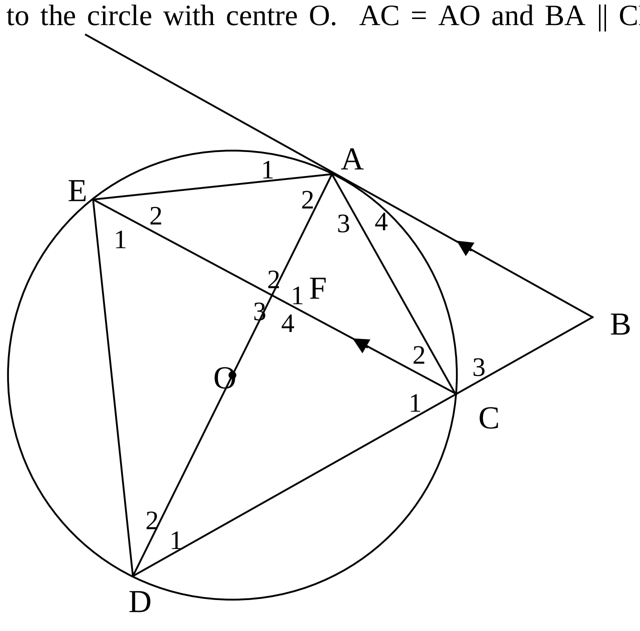
1.3.2 If HE = 2 cm, calculate the value of AD × HE. (2)

[8]

Question 2

2.1

In the figure below, AB is a tangent to the circle with centre O. AC = AO and BA || CE. DC produced, cuts tangent BA at B.



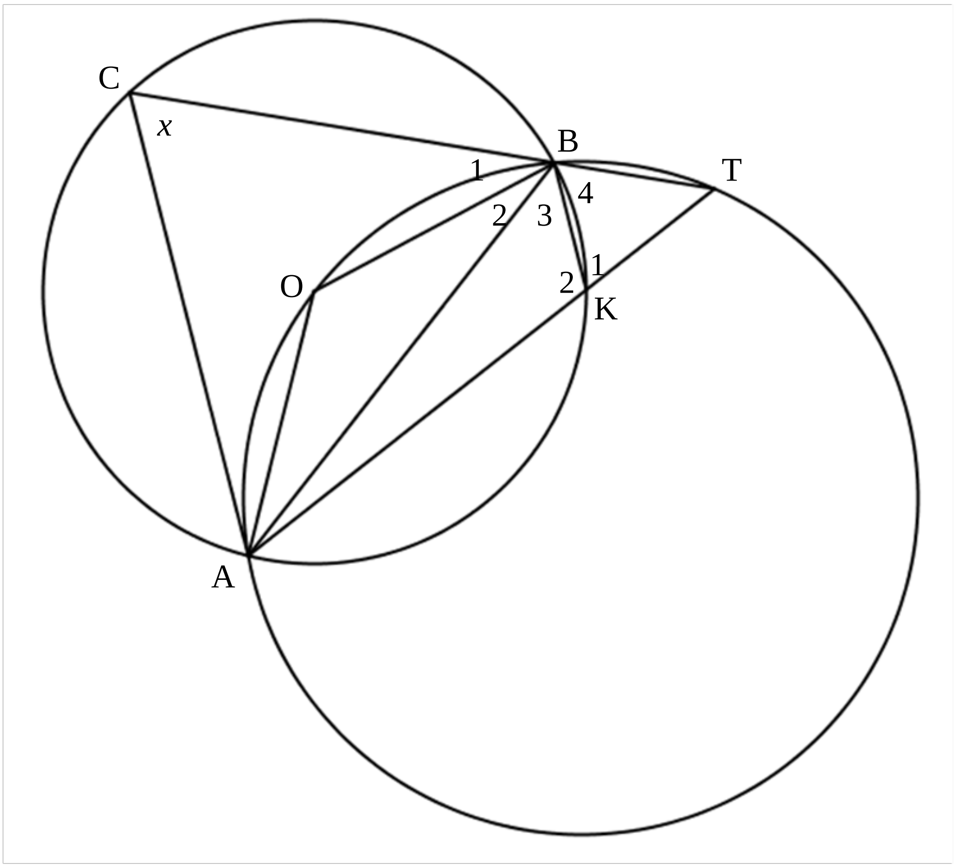
2.1.1 Show $\hat{C}_2 = \hat{D}_1$. (3)

2.1.2 Prove that ΔACF ||| ΔADC. (3)

2.1.3 Prove that AD = 4AF. (4)

2.2

O is the centre of the circle CAKB. AK produced intersects circle AOBT at T. $\hat{ACB} = x$



2.2.1 Prove that $\hat{T} = 180° - 2x$. (3)

2.2.2 Prove that AC || KB. (5)

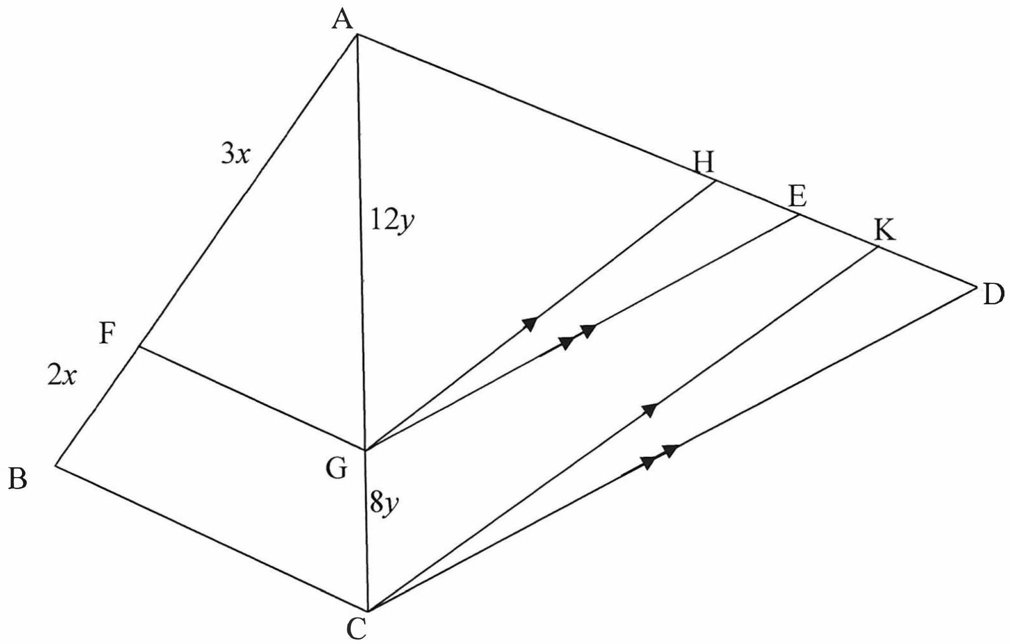
2.2.3 Prove that ΔBKT ||| ΔCAT (3)

2.2.4 If AK : KT = 5 : 2, determine the value of $\frac{AC}{KB}$ (3)

[14]

QUESTION 3

In the diagram, ΔABC and ΔACD are drawn. F and G are points on sides AB and AC respectively such that AF = $3x$, FB = $2x$, AG = $12y$ and GC = $8y$. H, E and K are points on side AD such that GH || CK and GE || CD.



3.1 Prove that:

3.1.1 FG || BC (2)

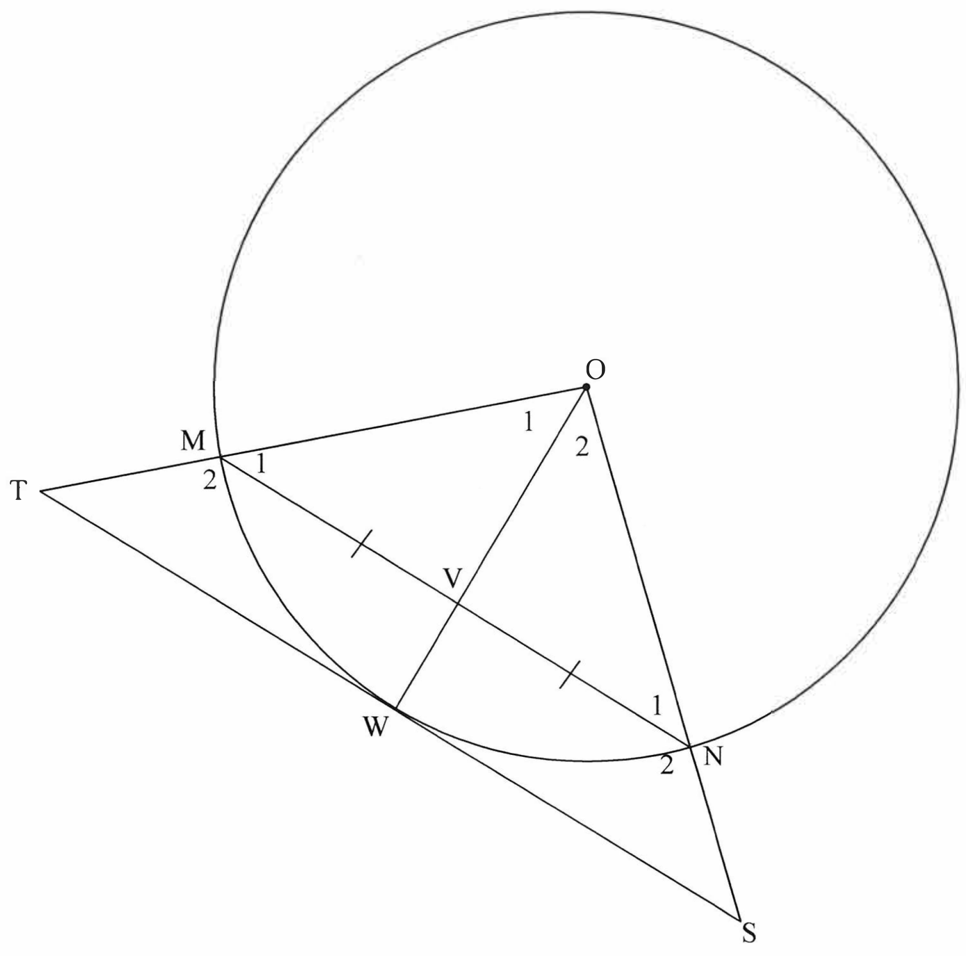
3.1.2 $\dfrac{AH}{HK} = \dfrac{AE}{ED}$ (3)

3.2 If it is further given that AH = 15 and ED = 12, calculate the length of EK. (5)

[10]

QUESTION 4

In the diagram, W is a point on the circle with centre O. V is a point on OW. Chord MN is drawn such that MV = VN. The tangent at W meets OM produced at T and ON produced at S.



4.1 Give a reason why OV ⊥ MN. (1)

4.2 Prove that:

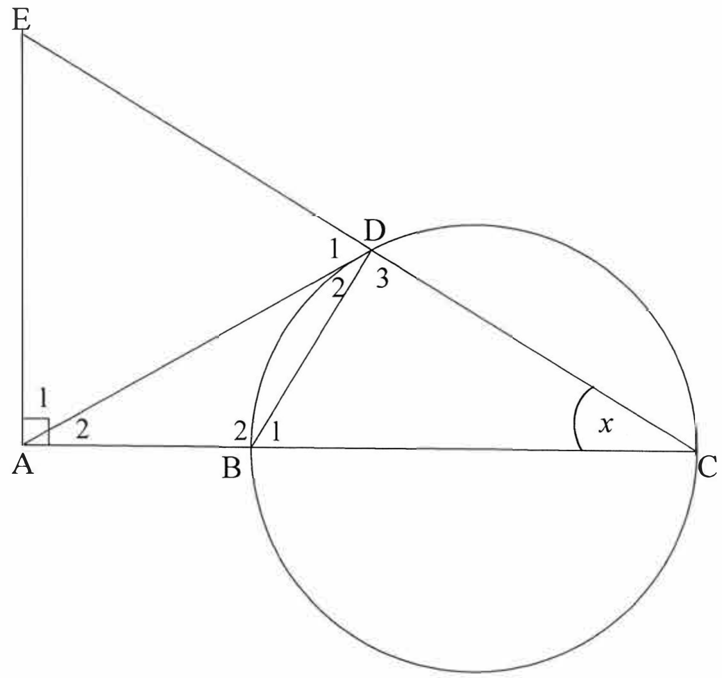
4.2.1 MN || TS (2)

4.2.2 TMNS is a cyclic quadrilateral (4)

4.2.3 OS . MN = 2ON . WS (5)

[12]

5. In the diagram, BC is a diameter of the circle. The tangent at point D on the circle meets CB produced at A. CD is produced to E such that EA $\perp$ AC. BD is drawn.
Let $\hat{C} = x$.



5.1 Give a reason why:

(a) $\hat{D}_3 = 90°$ (1)

(b) ABDE is a cyclic quadrilateral (1)

(c) $\hat{D}_2 = x$ (1)

5.2 Prove that:

(a) AD = AE (3)

(b) $\Delta ADB \ ||| \ \Delta ACD$ (3)

5.3 It is further given that $BC = 2AB = 2r$.

(a) Prove that $AD^2 = 3r^2$ (2)

(b) Hence, prove that ΔADE is equilateral. (4)

[20]